Chapter 3

Trends and Their Appraisal for Forecasting

The objectives of this and the next two chapters are:

1. To describe the four components of time series: trend, season, cycle, and irregular.
2. To show how to measure each of these components.
3. To test for success in selecting and calculating measures of trend and seasonal patterns.
4. To describe the use of trends and seasonal patterns in sales forecasting.

This chapter emphasizes trend extrapolation as a forecasting method, which assumes the pattern of change in the historical period will continue into the future.

3.1 Classical Time-Series Decomposition: Trend, Seasonal, Cyclical, and Irregular Components

The four components of traditional time-series analysis may be represented mathematically[1] as:

$$\underset{\text{original}}{O} = \underset{\text{trend}}{T} \times \underset{\text{season}}{S} \times \underset{\text{cycle}}{C} \times \underset{\text{irregular}}{I} \tag{3.1}$$

This expression uses a multiplicative relationship, which is highly useful because it allows for T (Trend) to be partially accounted for by population, price level, and other percentage growth phenomena. In the next chapter we illustrate that seasonal patterns are usually multiplicative in their effect.[2]

Here are detailed descriptions of the four components of time series:

1. The *trend* component, or secular trend, refers to gradual, long-term change. Abrupt shifts in trend may occur, usually due to observable reasons, but after such shifts gradual change will again appear.
2. The *seasonal* component consists of a recurring pattern within each year. Most seasonal patterns arise from weather and custom. Seasonal patterns typically change slowly over time.
3. The *cyclical* component refers to oscillations in the rate of business activity up and down about the trend. Business cycle movements occur for a variety of reasons, but by definition have no regularity in length or amplitude. Business cycles depend largely on the rates of business investment, the effects of government fiscal and monetary actions, and foreign influences.
4. The *irregular* component encompasses all fluctuations in a time series that do not have the characteristics of trend, seasonal pattern, or business cycle. Some irregular movements can be explained by unusual events in the economy, such as strikes, while others are minor economic fluctuations, almost random, with little hope of being predicted. The typical forecasting assumption is an average "zero irregular" component for such fluctuations.

3.2 Types of Trends

This discussion of the types of statistical trends emphasizes the economic appropriateness of each type and the implications for forecasting. We discuss five common types of trend that are helpful in preparing simple forecasts or revealing problems that will require more complex methods. These are:

1. Linear trend.
2. Combination of two or more linear or other types of trends.
3. Logarithmic or exponential trend.
4. Second-degree polynomial trend.
5. Second-degree logarithmic trend.

Fitting the economically appropriate type of trend, as well as a trend with good statistical fit, is important since the trend type affects error variation and, therefore, influences the conclusions to be made at later stages in the analysis of time series. Furthermore any errors made in choosing and calculating the trend are simply "pushed" into the measurement of the seasonal, cyclical, and irregular components, thus confusing the analysis.

Removing the trend from annual data[3] is accomplished by dividing original data by the trend. The model is:

$$\frac{O}{T} = \frac{T \times C \times I}{T} = C \times I = \text{cyclical and irregular components} \tag{3.2}$$

Removing trend from quarterly data is accomplished by:

$$\frac{O}{T} = \frac{T \times S \times C \times I}{T} = S \times C \times I = \text{seasonal, cyclical, and irregular components.} \quad (3.3)$$

Notationally we see that any errors in trend are forced into the residual measures of cycle and irregular in Equation 3.2 or into S x C x I in Equation 3.3.

3.3 Linear Trend: Freehand and Semiaverage Methods

Figure 3.1 shows annual retail sales of auto dealers in the United States. These dollar sales include the sales of both new and used cars as well as parts, repairs, services, and the like. The general upward movement indicates the rising importance of automobiles and their servicing in the national economy in addition to some inflation since sales are in current dollars.

For the eleven years of annual data, a linear trend adequately reflects the continuing upward movement at about the same average amount of increase per year. Figure 3.2 shows a linear trend fitted visually (or freehanded) to the auto sales data. The relationship of actual sales to trend indicates that sales in 1965 are moderately above trend, in 1967 below trend, and in 1970 appreciably below trend because of an auto workers' strike plus a decline in overall business activity. Actual sales in 1972 are well above trend reflecting: (1) recuperation from the strike, (2) higher levels of business activity, and (3) stimulus from the removal of excise taxes on automobiles.

Economic reasons for the upward trend include the gradually rising importance of autos in personal consumption patterns, population growth, and rising price levels. These last two influences may be removed by converting the sales data to a per capita basis, and further by dividing these figures by an appropriate price index to show per capita dollar sales in constant dollars. In some cases, the forecaster will make these adjustments first. In this introduction to sales forecasting, our purpose is only to recognize such elements of causation in an upward trend.

Freehand fitting of a straight line may provide good approximations to several mathematically fitted trends if you learn to apply visually the same criteria of goodness of fit that are applied by mathematical methods. The objective of fitting a linear trend freehanded is to adjust the height of a tentative line by trial and error until the sum of the vertical deviations above the line approximately equals the sum of the vertical deviations below the line. Then refine the slope of the line by adjusting its height in the left half of the data until the sum of the positive deviations approximately offset the sum of the negative deviations. Do the same for the right half of the data. Visual fitting of a straight line is done best with a transparent straightedge, which may be moved about to visually test for satisfying the aforementioned criteria.

We suggest that in freehand fitting of trend you should always first fit the straight line visually and then compare it with the calculated fit derived by the least-squares method (discussed later). Careful graphic analysis helps catch errors in calculations and is a vital part of the forecaster's reasoning process.

When inspecting any fitted trend you should concentrate on deciding whether the trend succeeds in portraying the average long-term nature of the change in the data. If the nature is not appropriately represented by a specific trend, then a different type of trend must be sought. Such different trends usually have important economic and statistical implications for the ultimate sales forecast.

The *semiaverage* method of linear trend fitting has four steps:

1. Divide the observations of a time series into equal halves. If the data contain an odd number of observations, include the center point in both the left half and the right half. This procedure slightly biases the height of the trend line toward the center observation because of its double weight, but the slope of the trend line will not be affected.
2. Calculate arithmetic means for both halves of the observations.
3. Plot each of these two arithmetic means at their respective vertical levels, with each average plotted horizontally at the midpoint of its time interval.
4. Draw a straight line through the two points.

The result of the calculation of a linear trend by the method of semiaverages is shown in Figure 3.3 where the semiaverage trend differs little from the freehand trend in Figure 3.2. The main reason for presenting the semiaverage method is to illustrate the mechanics underlying good visual freehand fitting. The concept of semiaverage trend is worthwhile primarily as a method for improving freehand fitting. The semiaverage method is never used for computer calculation, however, because the least-squares method described next is universally accepted and is an integral part of statistical forecasting methodology.

3.4 Linear Trend: Least-Squares Method

Least-squares linear trend is the most commonly used statistical trend because it has all of the characteristics of an "average" trend plus other mathematical advantages. The reason for using a calculated trend instead of a freehand trend is for objectivity, providing a method whereby two persons can arrive at identical trends. The method of least-squares, then, gives a logically reproducible, unique solution. The equation for linear trend is:

$$Y_c = a + bX. \quad (3.4)$$

Linear trend determined by least-squares is based on two criteria:

1. The sum of the algebraic deviations of the actual data from the trend line is zero, i.e., the positive deviations cancel the negative ones, defining an average trend through the observations.
2. The sum of the squared deviations from the trend line is a minimum, giving rise to the term "least-squares."

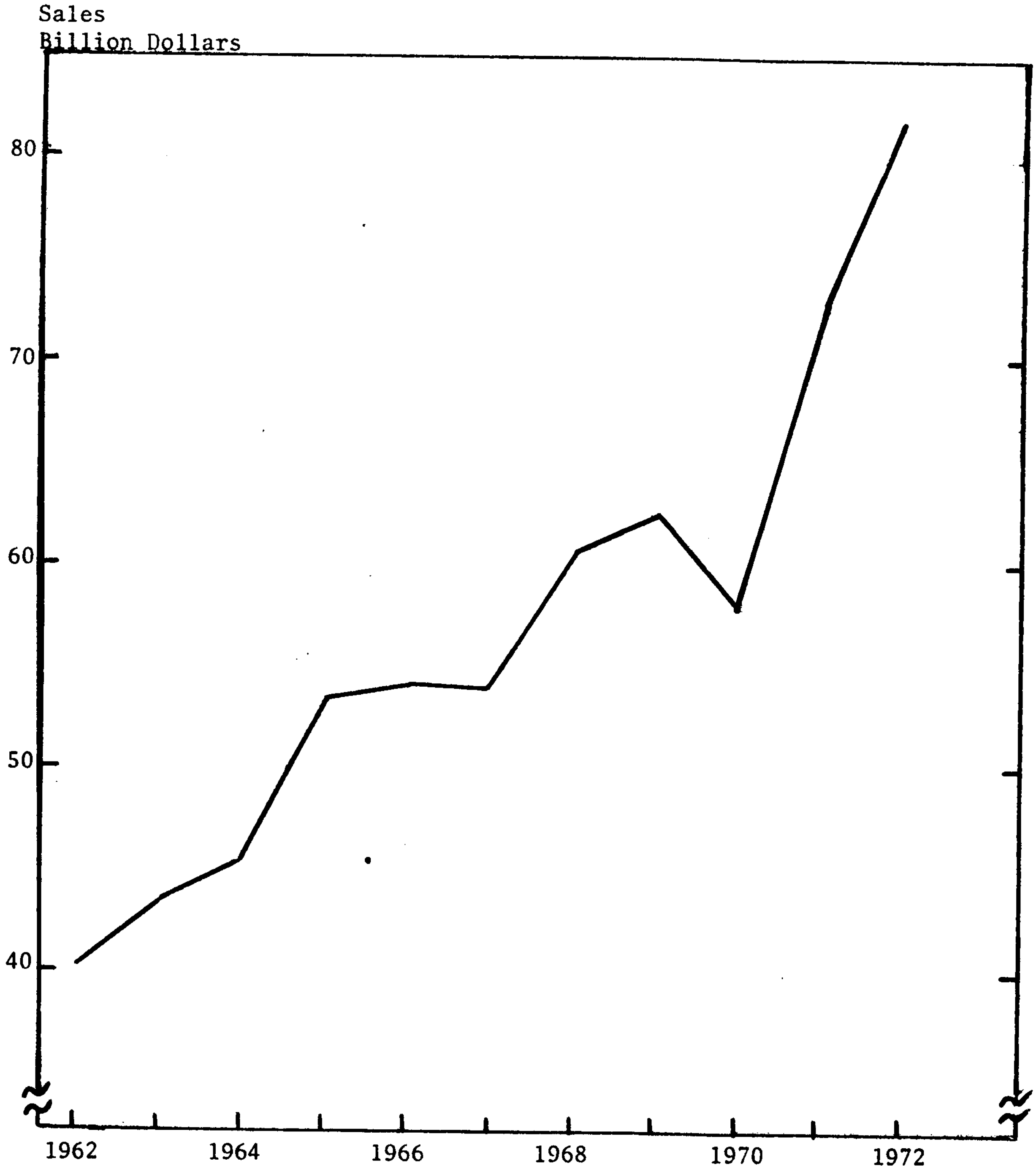
Notice that criterion 1 sets the height of the least-squares line, while criterion 2 determines the slope.

An example of manual calculation of a least-squares linear trend is shown in Table 3.1 using coded "x" for years, so that the sum of the x's or Σx, equals zero. For an odd number of observations, the center year is assigned an x of zero. From this new centered origin, future years are

Figure 3.1

Auto Dealer Retail Sales

United States Industry Annual Totals, Current Dollars



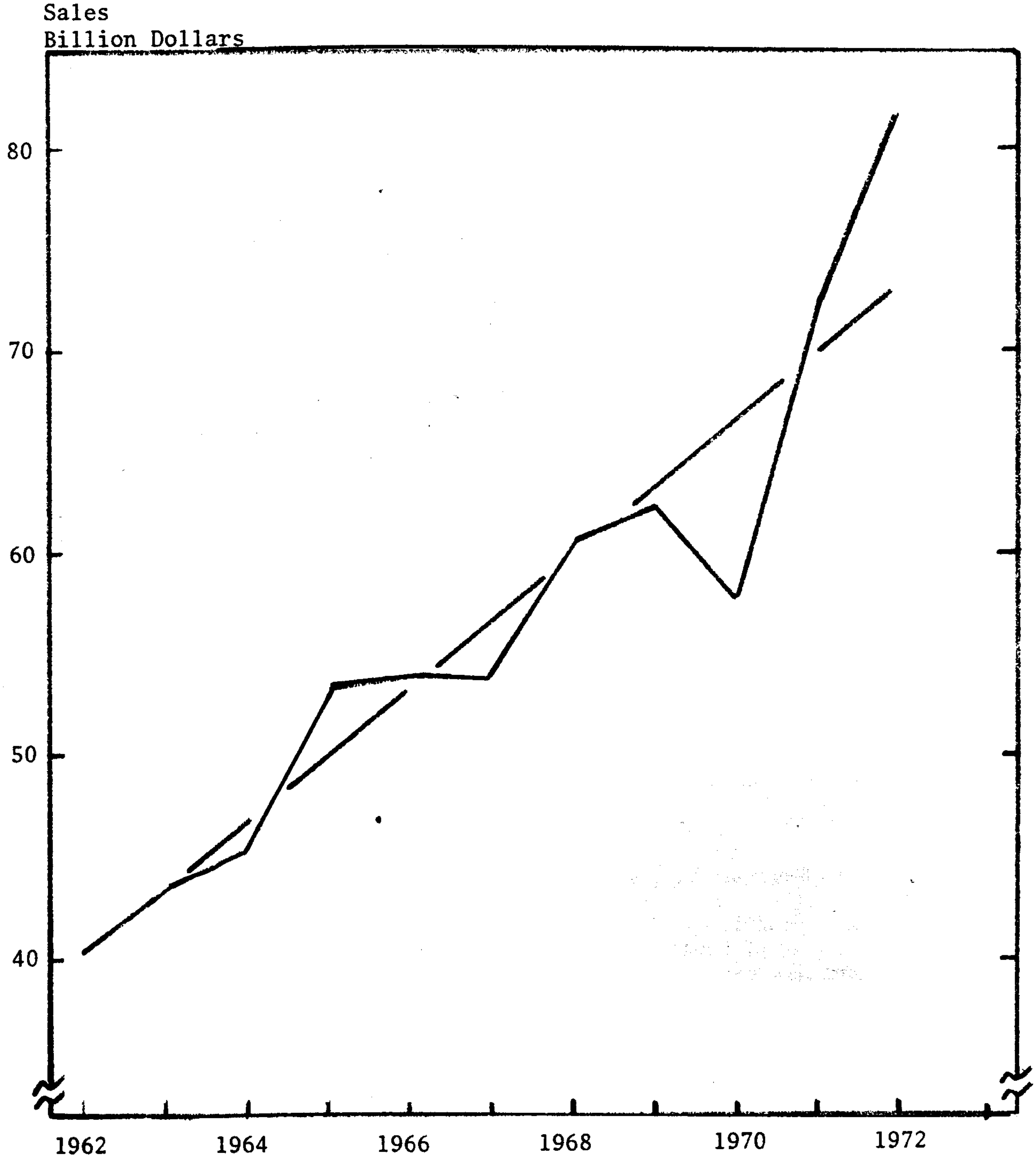
Source: Table 3.1

Figure 3.2

Auto Dealer Retail Sales

United States Industry Annual Totals, Current Dollars

Linear Trend: Freehand Method



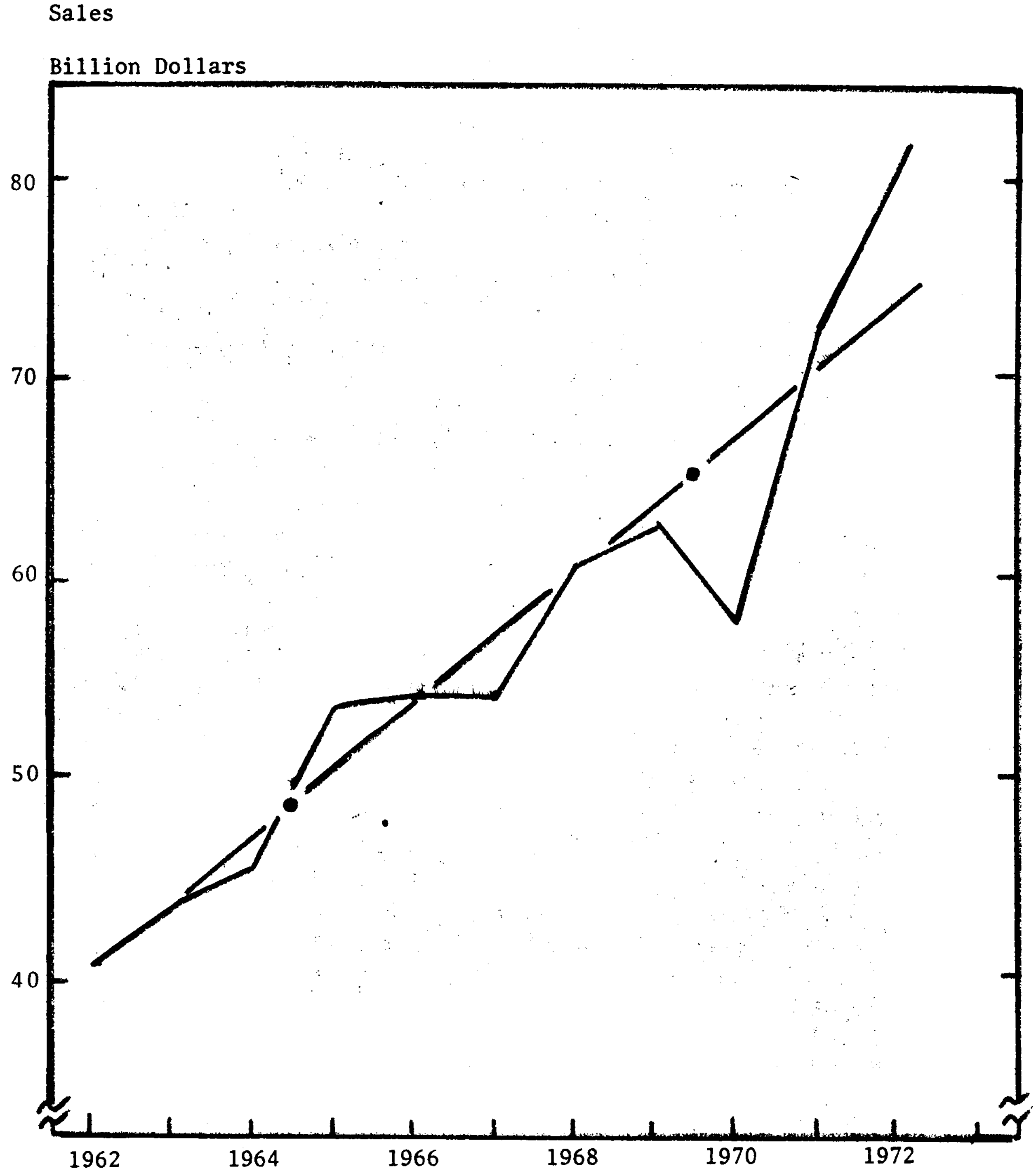
Source: Table 3.1

Figure 3.3

Auto Dealer Retail Sales

United States Industry Annual Totals, Current Dollars

Linear Trend: Semiaverage Method



Source: Table 3.1

Table 3.1

Auto Dealer Retail Sales

United States Industry Annual Totals, Current Dollars

Linear Trend: Least-Squares Method

(1) Year	(2) Y Sales Bil. Dol.	(3) x Coded Year	(4) xY	(5) x^2	(6) Y_c Sales Trend Bil. Dol.
1962	40.1	–5	–200.5	25	39.3
1963	43.2	–4	–172.8	16	42.8
1964	45.6	–3	–136.8	9	46.4
1965	53.5	–2	–107.0	4	49.9
1966	54.1	–1	–54.1	1	53.4
1967	54.0	0	0	0	56.9
1968	60.7	+1	+60.7	1	60.4
1969	62.4	2	+124.8	4	63.9
1970	58.3	3	+174.9	9	67.5
1971	72.6	4	+290.4	16	71.0
1972	81.5	5	+407.5	25	74.5
	626.0 ΣY	0 Σx	+387.1 ΣxY	110 Σx^2	626.0 ΣY_c

$$a = \frac{\Sigma Y}{n} = \frac{626.0}{11} = \$56.909 \text{ bil.} = \overline{Y} = \text{height at } x = 0$$

$$b = \frac{\Sigma xY}{\Sigma x^2} = \frac{+387.1}{110} = +\$3.519 \text{ bil.} = \text{slope} = \text{average increase per year}$$

$$Y_c = a + b(x)$$

$$Y_c = 56.909 + 3.519(x) = \text{trend equation}$$

$$Y_c(1973) = 56.909 + 3.519(6) = \$78.0 \text{ bil.}$$

Source: Survey of Current Business and Business Statistics.

numbered +1, +2, +3, . . . , and years progressing back in time are numbered −1, −2, −3, ... Here the coded x equals one year. Such coding of time as x allows the use of simplified hand computational formulas for slope and intercept.

When an even number of years appears, then the two center years are coded +1 and −1, and subsequent numbering for future years is +3, +5, . . . , while going backwards is −3, −5, . . . from the beginning. In this case the coded x equals ½ year. This method is shown in detail by Table 3.2 (discussed later).

Having fitted the linear trend, in Table 3.1 and Figure 3.4, we must appraise whether the trend is appropriate from economic causation and business cycle standpoints. Since the trend in Figure 3.4 is nearly identical to the freehand trend of Figure 3.2, the same conclusions apply, namely that the least-squares trend is a moderately good representation of the average annual change in the series.

If we consider a projection of this trend as a preliminary forecast for 1973, as shown in Figure 3.4, we imply that 1973 will be cyclically down from 1972. Most forecasters were not saying this at the end of 1972 for 1973. We must be careful, therefore, to accept the trend at this point only as a trend and plan to make a separate analysis later of the forecast for cyclical position in 1973 relative to trend.

Traditional economic trend analysis usually requires at least twenty annual observations for proper fitting. In applied sales forecasting, however, data for a consistent product service definition may not be available for such lengthy periods. Instances with annual data for only ten years are quite common. Therefore, in short-term sales forecasting for five to eight quarters ahead, ten years of data are usually viewed as adequate, practically speaking. We caution that for long-term sales forecasting of five to twenty years ahead, however, historical data for at least twice as many years as are being projected are desired to reflect long-term trend, changes in trends, and a matching of the economic reasons for changes in statistically observed trends.

3.5 Changes in Linear Trend

Changes in the slope of trend, abrupt shifts in level of trend, and combinations of two or more types of trend frequently are necessary to reflect major changes in economic and business conditions.

Figure 3.5 illustrates a trend calculation for a household commodity group. It shows annual dollar sales of Safeway Stores Incorporated, a large company engaged in the retail chain grocery business. The plotted data show a moderately consistent linear trend for 1961 through 1968, but 1969 through 1972 reflects a sharply steeper trend.

Visual analysis of Figure 3.6 indicates that a single straight line trend fitted to Safeway's data is a poor fit, because the beginning and ending years are consistently above the trend and the middle years are consistently below it. This illustrates a failure to fit a trend that appropriately represents the long-term nature of the data. The reason for this misfit is that the deviations of actual data from the trend reflect a *systematic* pattern of error or a combination of random error and business cycle deviations from trend that correspond to knowledge of general cyclical conditions in the grocery industry.

We previously noted the change in the Safeway trend in 1969. One approximation to this change appears in the two linear trends in Figure 3.7. The 1968 point is a convenient data point common to both trends. Such a point will not always be available, as is the case here, but a common point is not necessary so long as the first trend moves smoothly into the second.

Inspection shows that the two trends fairly represent two different rates of absolute annual change. The recent trend is based on only five observations, which is a very small number from a statistical standpoint. The lesson is: a trend fitted to only a few observations must fit well to be useful.

Use of two different trend lines must be logically justified in business or economic conditions that can be observed and eventually measured. In this case the justification is found in corporate reports. Safeway started on an expansion-acquisition program in the mid-sixties with results appearing in their sales data beginning in 1968 and very clearly by 1969 and 1970. Further examples of changes in trend, or combination of trends, are presented in subsequent discussion. Regardless of the type of trend, the fitted line has to be a good fit, and adequate causation must be determined for changes or shifts.

3.6 Logarithmic Trend

Logarithmic, or exponential, trends describe time series with constant percentage rates of change over time. Such trends fit many human and animal population series where biological reproduction tends to establish a geometric progression. Many price indexes follow logarithmic trend where a constant percent increase each year has become the pattern. A logarithmic trend exemplifies one kind of curved or nonlinear trend in that it has a changing absolute amount of change per time period.

Plotted on semilogarithmic graph paper[4] a logarithmic trend appears as a straight line. Such a trend is calculated by determining the logarithm of each observation and then fitting a least-squares linear trend to the logarithms.[5] The logarithmic trend line has this equation:

$$\log Y_c = \log a + X \log b \qquad (3.5)$$

The *log a* for logarithmic trend calculations determines the height of the trend line at the time period of the centered arbitrary origin. Since *log a* is the average of logarithms, then *a* is the geometric mean of the original observations. Since *log a* is centered by this method, its time location is at the arbitrary origin, corresponding to *o* in the coded x's.

The *b* is the geometric mean of the ratios of each observation to its preceding observation. These ratios, called "slope ratios," reflect average percent increase per time period for the series.

Logarithmic trend values are determined by calculating Y_c values from Equation 3.5. To be meaningful, the resulting logarithms must be converted to antilogs. When plotting on semilogarithmic scales, two observations are adequate for determining the linear logarithmic trend. To plot on an arithmetic vertical scale, all Y_c values must be calculated and plotted.

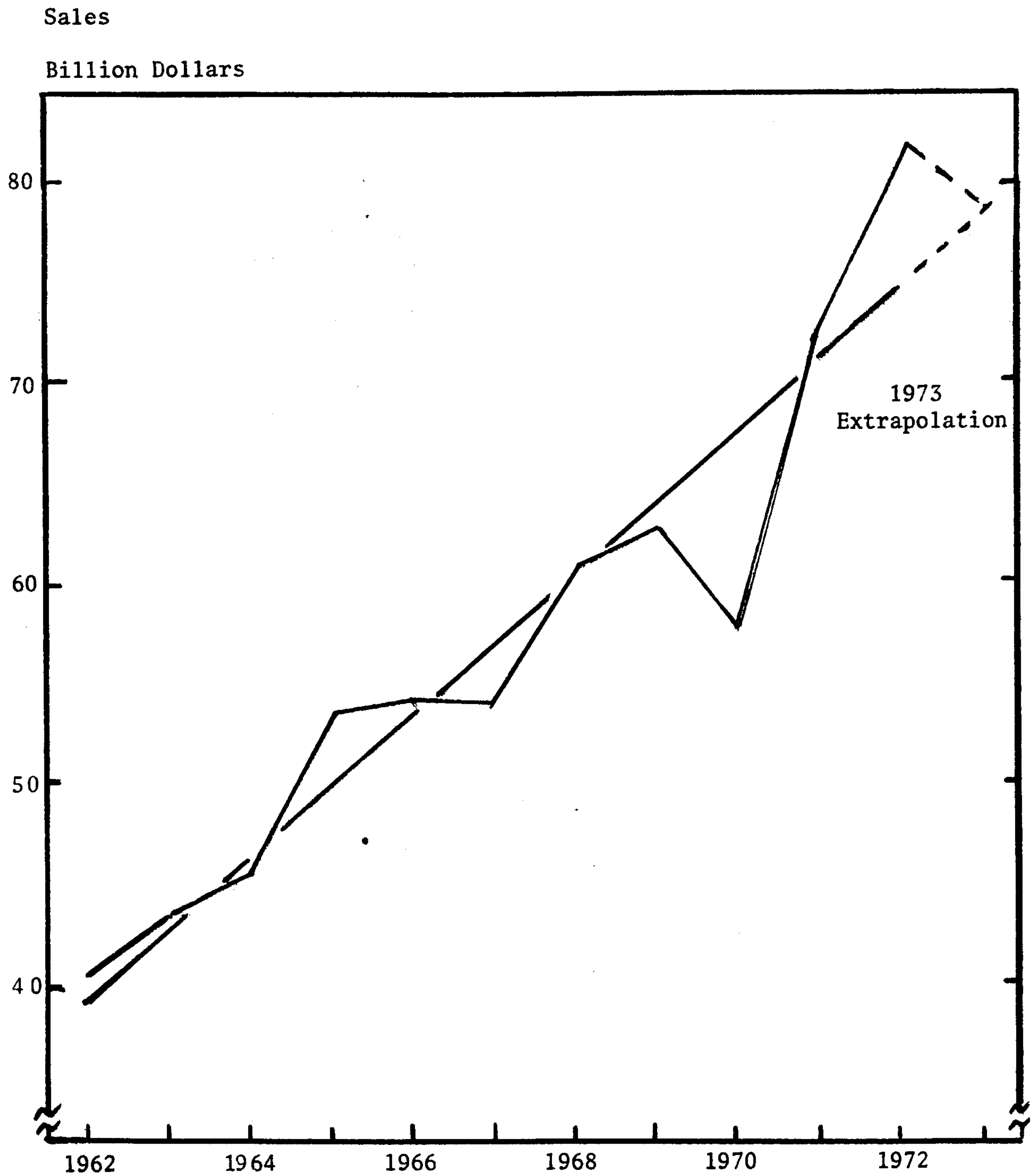
The original data and the resulting logarithmic trend for

Figure 3.4

Auto Dealer Retail Sales

United States Industry Annual Totals, Current Dollars

Linear Trend: Least-Squares Method

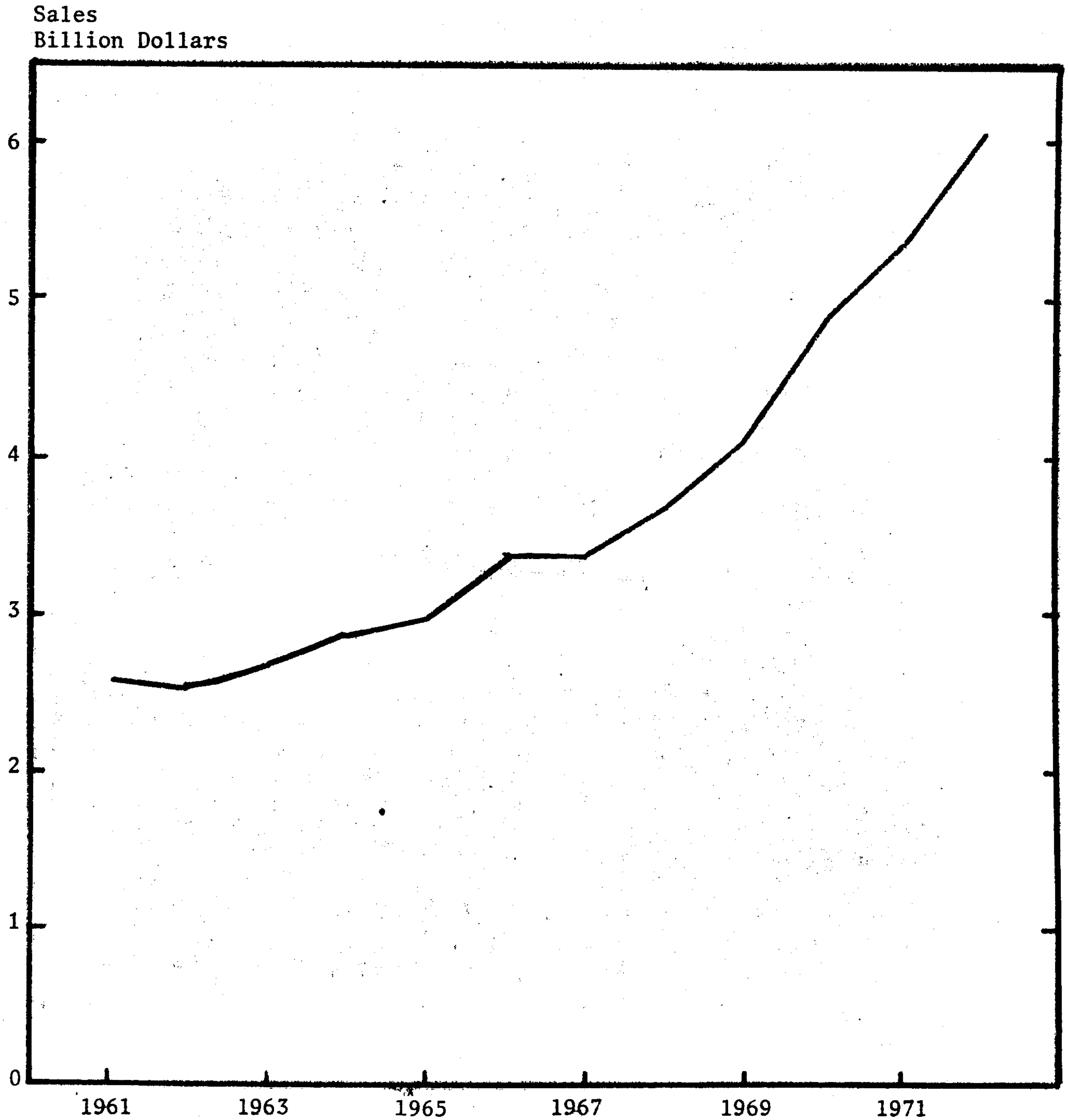


Source: Table 3.1

Figure 3.5

Safeway Stores Incorporated

Annual Sales, Current Dollars



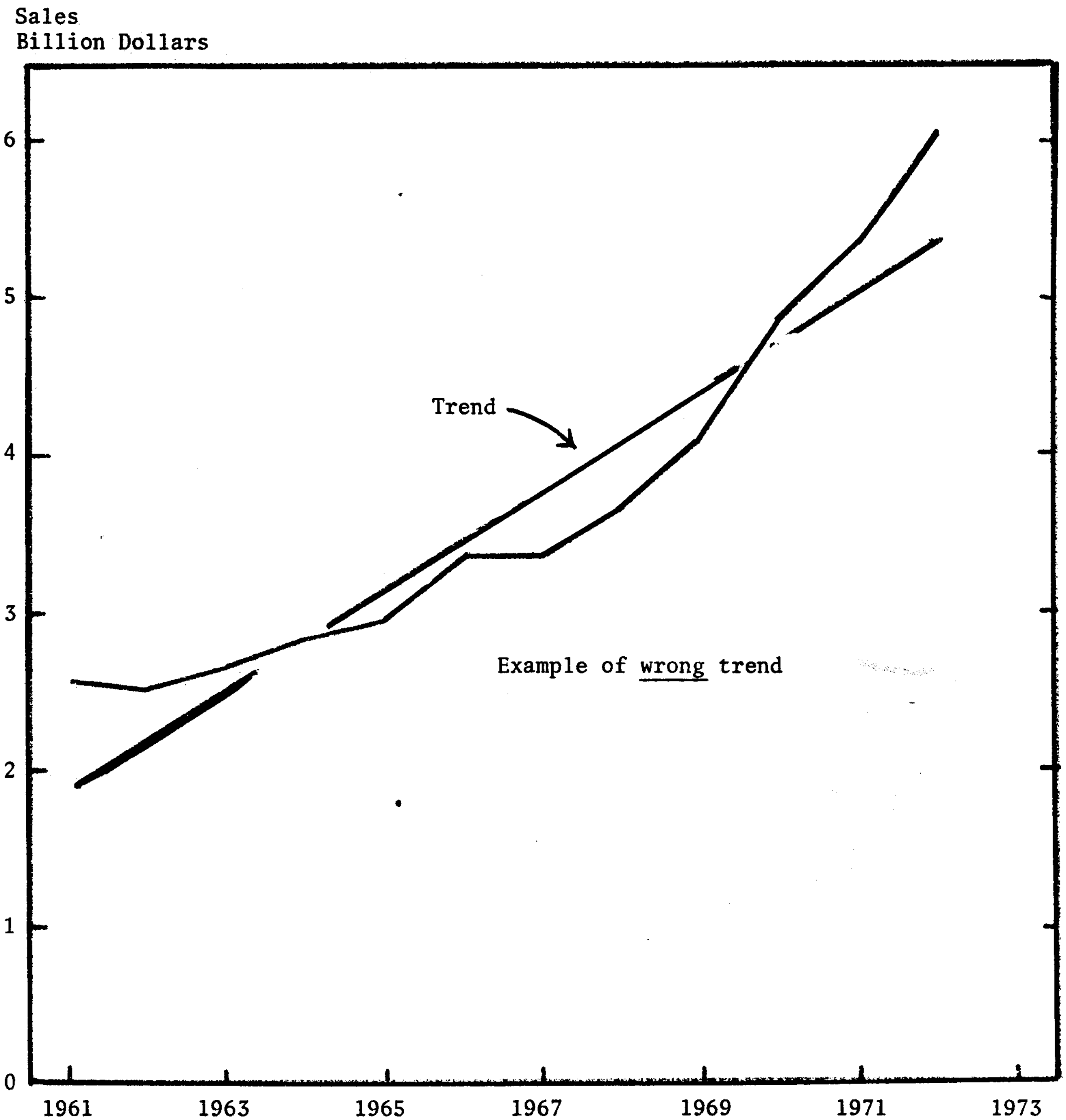
Source: Safeway Stores Incorporated

Figure 3.6

Safeway Stores Incorporated

Annual Sales, Current Dollars

Linear Trend: Least-Squares Method



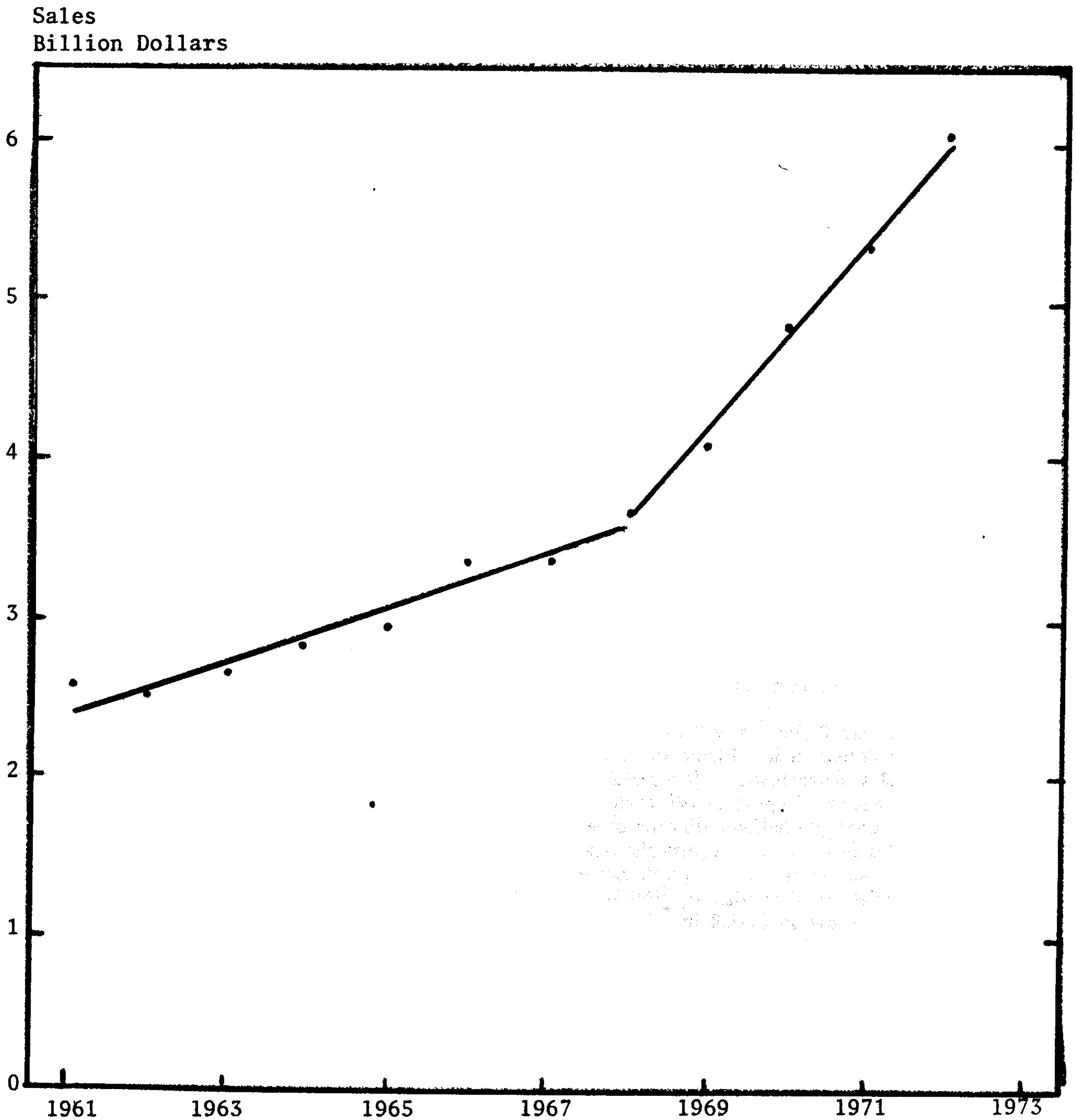
Source: Safeway Stores Incorporated

Figure 3.7

Safeway Stores Incorporated

Annual Sales, Current Dollars

Two Linear Trends: Least-Squares Method



Source: Safeway Stores Incorporated

the U.S. Electric Production Index from 1954 through 1972 appear on an arithmetic scale in Panel A of Figure 3.8. The fitted trend is close to the original data everywhere and shows no systematic pattern of error. We conclude that a log trend is a good statistical fit for the electric utility industry in the United States. Panel B of Figure 3.8 shows the same original data and logarithmic trend on a logarithmic vertical scale.

For these electric utility data, a, the geometric mean is 74.8 centered at the arbitrary 1963 origin. We note that 74.8 is fairly close to the actual 1963 index of 73.6 (1967 = 100). The average slope ratio for the electric production index is 1.079, reflecting a 7.9 percent average increase per year for the series.

The underlying causes of change in this series are primarily population growth and a steady increase in per capita consumption of electricity. Both of these causes are inherently more akin to constant percentage change than constant absolute amount of change. In our example, therefore, the mathematical form of the trend function is related to the nature of the two main causal factors. Identifying this relationship is a desirable goal in trend fitting, though it is not always as well accomplished as in this case.

Is the extrapolation of trend useful as a good final forecast or only as a kind of preliminary forecast? The answer to this question depends not only on whether past forces for change will continue at the same rate in the future but also on whether any extreme pressures for increase or any new limitations on increase will appear.

Our judgment is that in the forecast for 1973 and 1974, population increase and per capita desires to increase electricity usage will continue at slightly under the recent rate, and the ecological and other restraints on new power plant construction will also begin to cause a slower increase in electric capacity. These reasons probably will combine to cause the 1974 index of electric utility production to be slightly below the logarithmic trend extrapolation, possibly at a 6.5% growth rate, yielding an index forecast of 167 for the year. Readers after 1974 will be able to make their own appraisal of this forecast.

A logarithmic trend may decline as well as increase. After many periods of decline, a logarithmic trend will always approach zero. This characteristic is often useful, as in describing the declining phase of a product's life cycle.

The logarithmic time trend plotted on arithmetic paper is a curved trend. But this curved trend is a particular type of curve that will not necessarily fit any gentle curve appearing on paper. The Safeway data originally plotted on Figure 3.5, for example, show an overall increase with larger annual amounts of advance in recent years than in early years. It is possible that the trend may be represented by a straight line on semilogarithmic paper, indicating a constant percent of annual increase. In contrast, a straight line on arithmetic paper represents a constant absolute amount of increase each year.

To test for logarithmic trend graphically, we plotted Safeway sales in Figure 3.9 with a logarithmic vertical scale, and fitted a least-squares logarithmic trend. This straight line does not adequately portray the nature of change in these data because the actual points at the beginning and the end of the period are consistently above the straight line logarithmic trend, while most of the points in the center are below the trend. This is a *systematic* pattern of *error* and means that the constant rate of increase of logarithmic trend is not a good description of the basic progression of the Safeway data.

Figure 3.10 shows two logarithmic trends fitted to the Safeway data plotted on semilogarithmic scales. The two trends are appropriate visual fits for the two periods, but these two constant rate-of-change trends are not significantly better than the two constant amount-of-change trends on arithmetic scales of Figure 3.7.

The choice between the two arithmetic trends in Figure 3.7 and the two logarithmic trends in Figure 3.10 is made on the basis of (1) the uses to be made of the trends and (2) the subjective appraisal of how well each type of trend logically explains the underlying causes of change. If Safeway, for example, intends to continue expansion efforts at about a constant percentage each year, then a logarithmic trend is clearly appropriate. The log trends have the advantage of yielding annual growth rates, which are convenient descriptors. Of course, other conditions might call for different trend types.

3.7 Shifts in Levels of Trend

Two kinds of changes in time trend have been illustrated:

1. Two straight lines at different average annual amounts of change (Figure 3.7).

2. Two logarithmic trends at different average annual percentages of change (Figure 3.10).

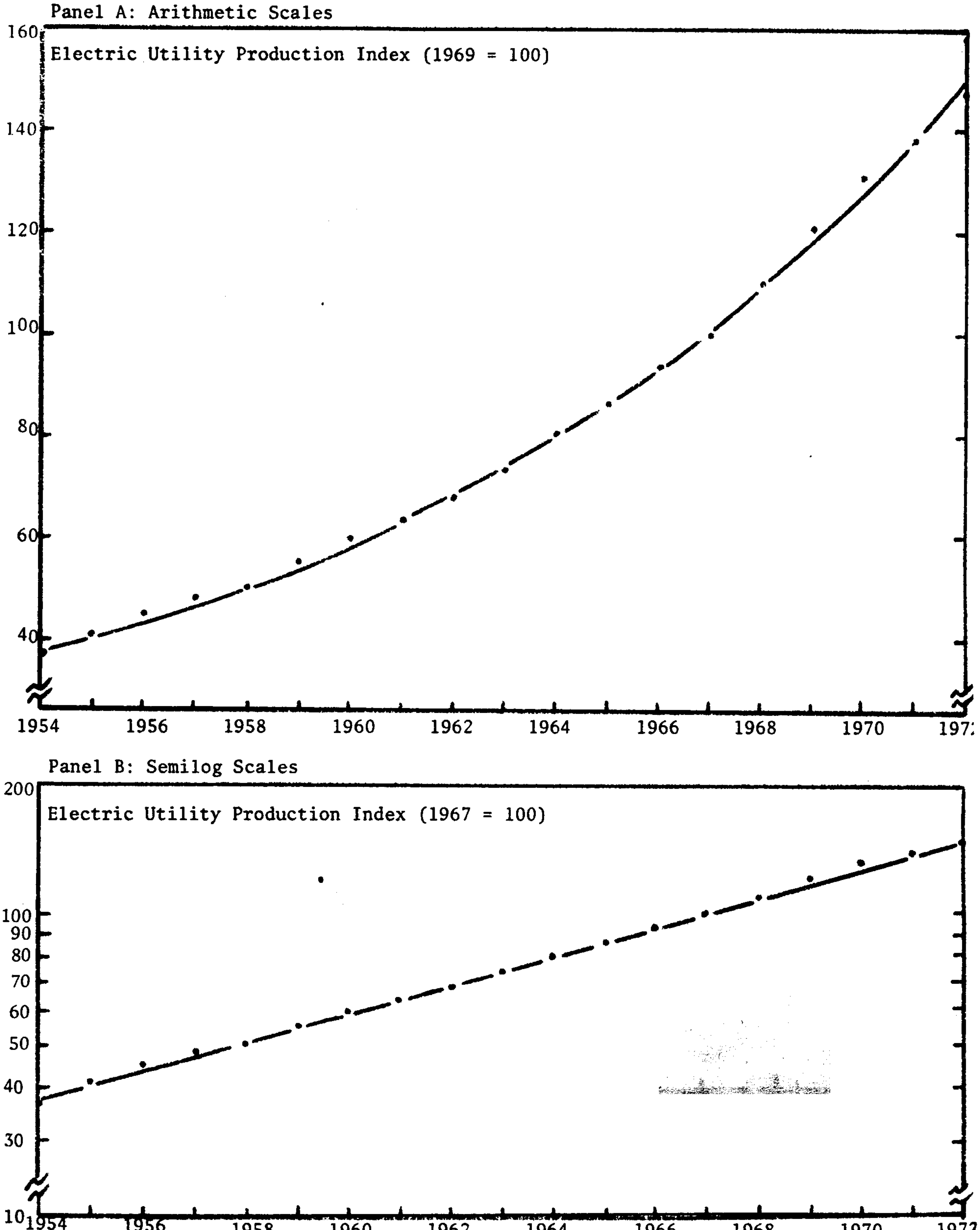
A different problem appears in Figure 3.11 for Style Shop (a pseudonym), a men's clothing specialty store located in a major city of the Southwest. Notice that this is an individual company series as contrasted with the several previous industry series. This series exhibits a larger than usual increase in 1966, followed by a very large increase in 1967, and then by increases in 1968-1972 of the same order of magnitude as those prior to 1965.

A linear trend is shown in Figure 3.12, but it does not adequately represent the underlying nature of change for the series, again, because of systematic error. Sales from 1961 through 1966 are all below the trend, and sales from 1967 through 1970 are all above it. To accept this trend we would have to verify from conditions peculiar to Style Shop's area of business that the implied cyclically low and cyclically high periods just mentioned are characteristic of the men's clothing market in this market area. While we are not experts in men's clothing, we suspect that such a cyclical explanation here is inappropriate.

Figure 3.13 shows a linear trend from 1957 through 1965 and a separate higher level linear trend from 1967 through 1972. Statistically, this is a reasonable fit, although other types of curves might be tried.

The important issue is whether a viable explanation exists for the major change in economic conditions, essentially a change within the market structure for Style Shop, that would justify shifting from the end of one trend in 1965 to a new higher level starting in 1967. In this case, a major change did occur. At this time men's clothing styles changed drastically from the conservative dark suit with

Figure 3.8
Electric Utility Production Index
United States, 1967 = 100
Logarithmic (Exponential) Trend: Least-Squares Method



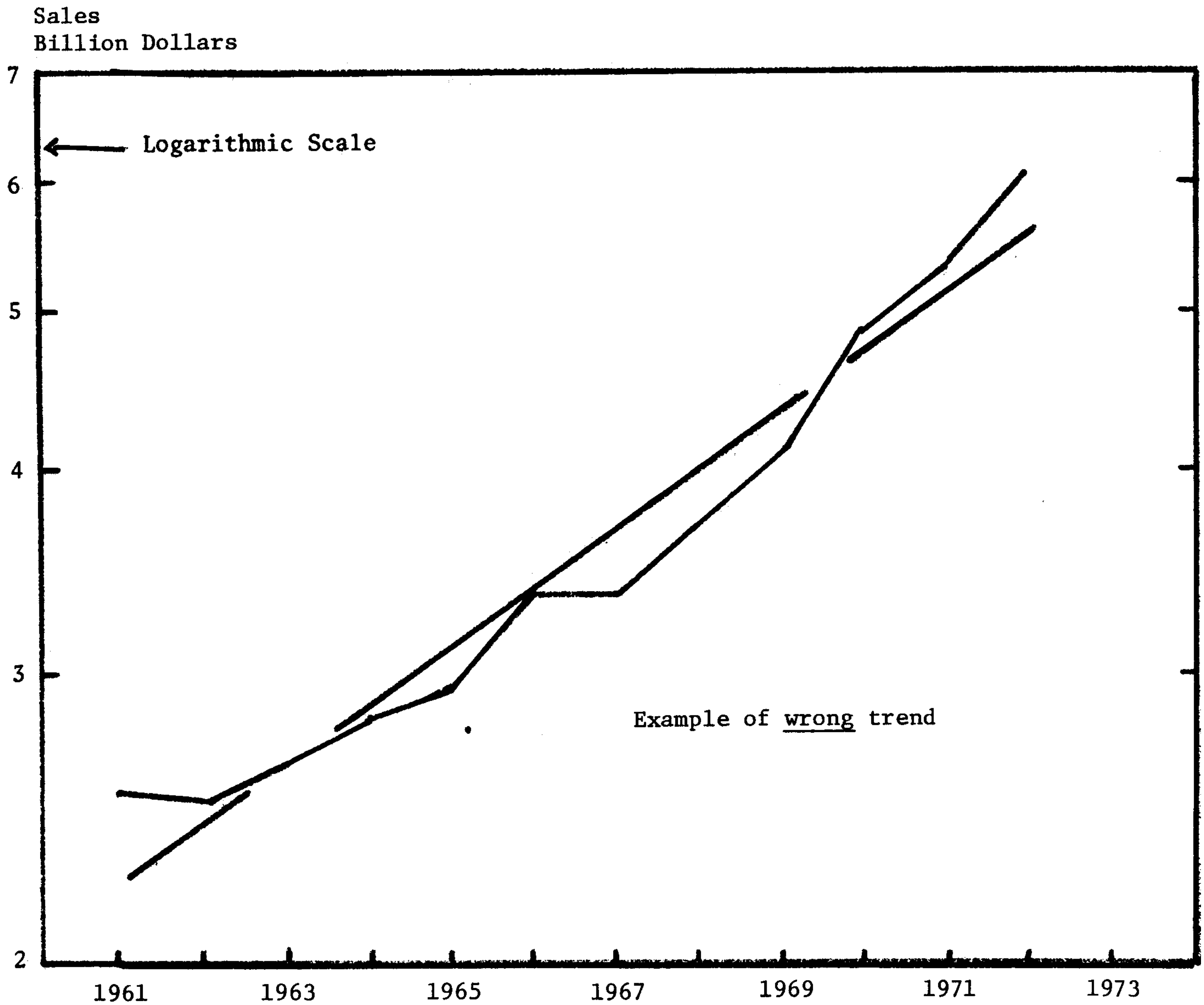
Source: Business Statistics; Survey of Current Business

Figure 3.9

Safeway Stores Incorporated

Annual Sales, Current Dollars

Logarithmic (Exponential) Trend: Least-Squares Method



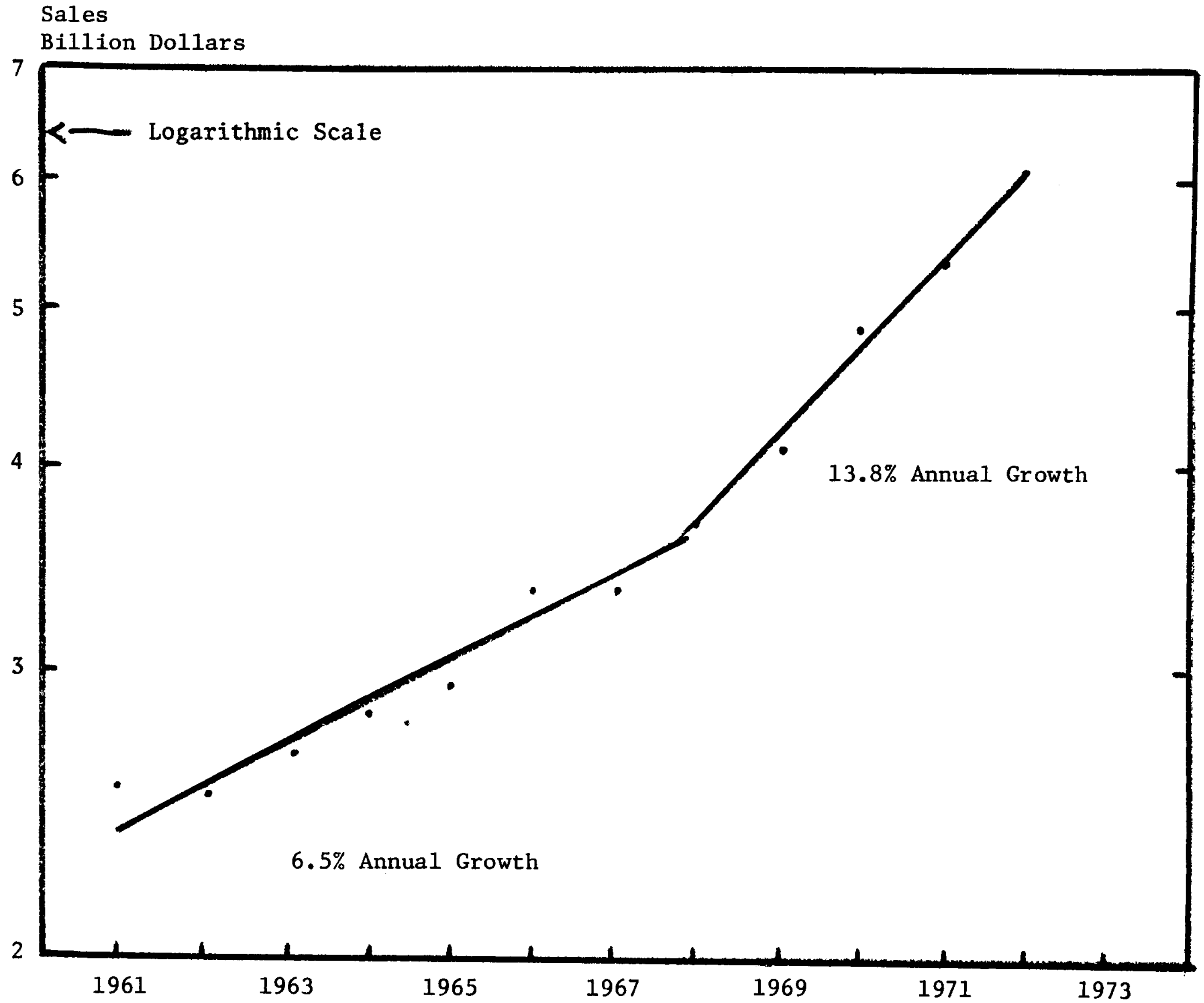
Source: Safeway Stores Incorporated

Figure 3.10

Safeway Stores Incorporated

Annual Sales, Current Dollars

Two Logarithmic (Exponential) Trends: Freehand Method

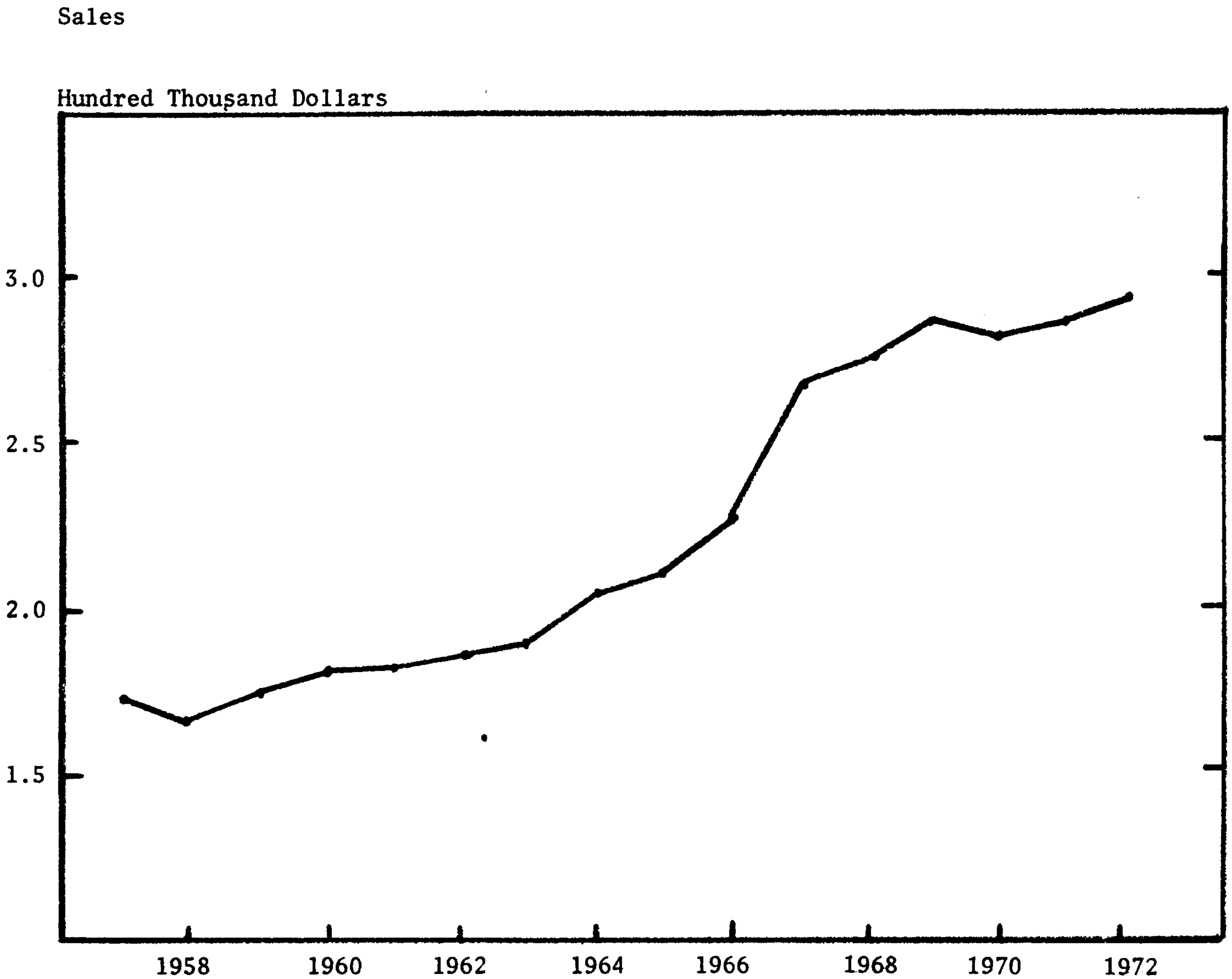


Source: Safeway Stores Incorporated.

Figure 3.11

Style Shop*

Annual Sales, Current Dollars



* Pseudonym

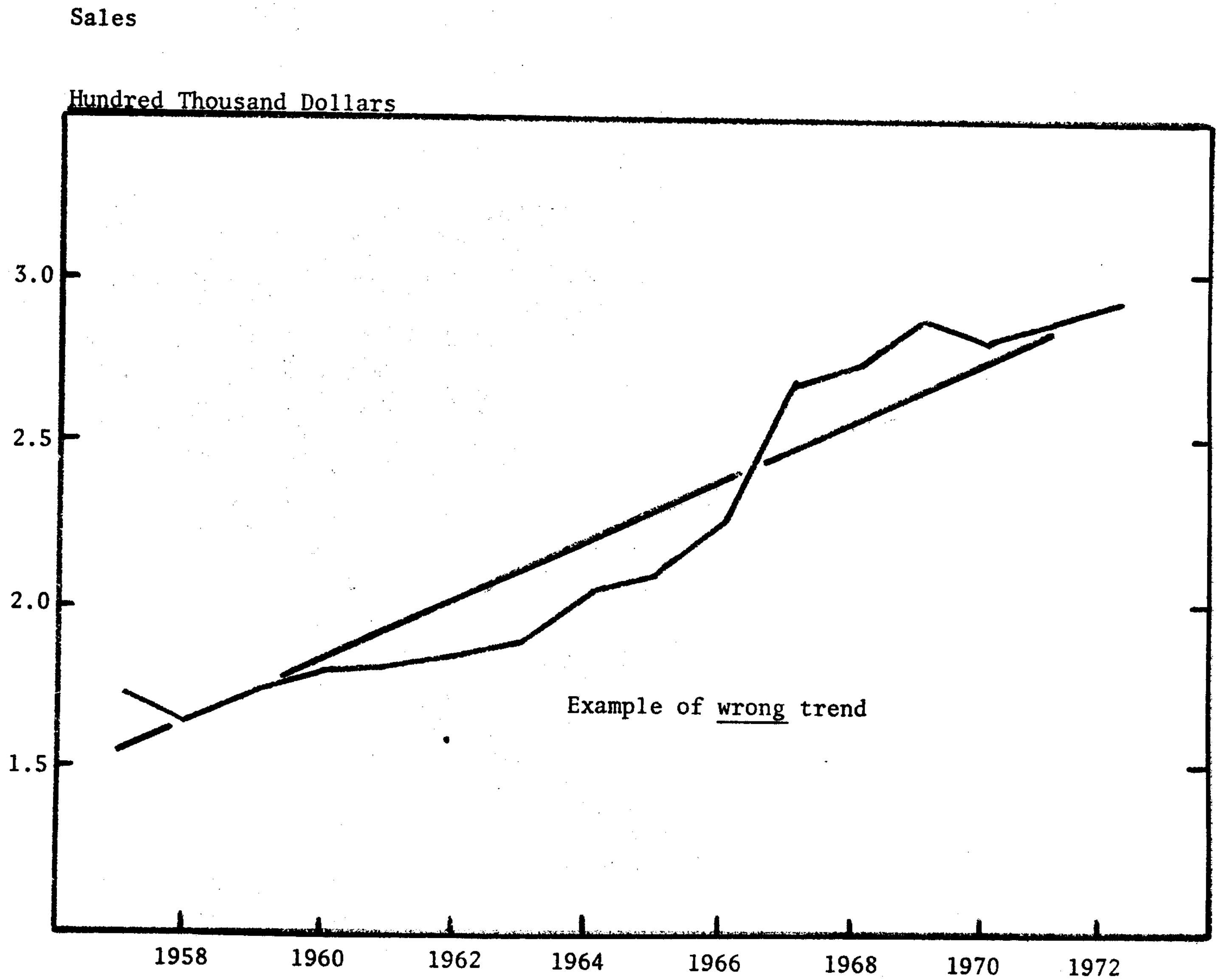
Source: Case study of actual data.

Figure 3.12

Style Shop

Annual Sales, Current Dollars

Linear Trend: Least-Squares Method



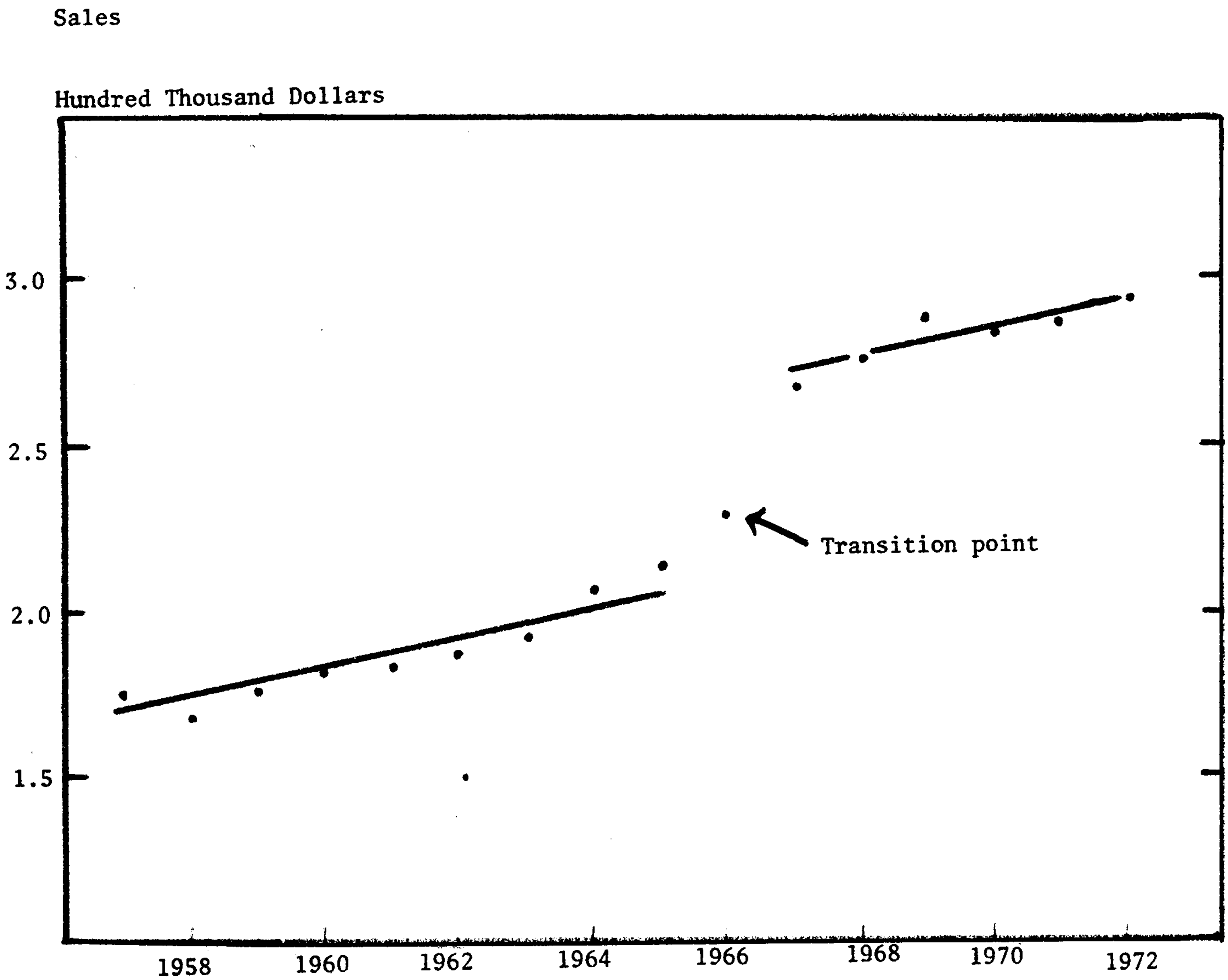
Source: Case study

Figure 3.13

Style Shop

Annual Sales, Current Dollars

Two Linear Trends: Least-Squares Method



Source: Case study

narrow lapels, narrow ties, and white shirts to brighter colored suits with wide lapels, wide ties, and colored shirts, all in bolder patterns than previously shown or purchased for popular demand. The year 1966 appears to be the transition point in this process, and for this reason it is omitted from consideration when trend fitting.

From the standpoint of Style Shop's economics, the 1966-1967 trend change represents a substantial shift, after which, the store's sales pattern resumes its linear trend at about the same annual absolute increase as in the 1957-1972 period. For forecasting purposes, if we assume that men's clothing styles will continue at the current level of color, variety, and style change, then an extrapolation of the 1967-1972 trend is a good trend forecast. Short-term cyclical variations around this trend, however, should be expected.

The possibility also exists that a future major change in the demand for men's clothing could occur in a downward direction. This possibility might cause a third trend to appear after a transition period. This third trend would be at a lower level than the 1967-1972 trend but probably not as low as the 1957-1965 trend. A conceivable economic explanation for such a drop is that the current styles will continue, but that men have accomplished the major replacement of their long-wear clothing from the old style to the new style and that few innovations will be accepted by consumers. If this assessment is correct, then men will continue wearing their "wide-lapel era" clothes for some time and spend less on annual purchases, thus creating the lower prophesied trend.

The foregoing prognostication illustrates the process of combining statistical methods with economic causation and knowledge of Style Shop's market. We emphasize that statistical methods never prove economic or business causation, but rather provide a means of quantifying the nature and direction of change. The type of trend equation must conceptually fit the economic description of events occurring within the area being studied.

Our speculation on the causes for the quantum trend shift in Style Shop's sales of men's clothing was confirmed by consultation with the store owner and with four experienced national executives in the men's wear industry. These experts substantiated that during 1966 a major transition in the industry had taken place. Part of this transition was deliberately introduced by stylists and designers, but the clothing experts emphasized that male consumers seemed ready for the change. They stressed that only this receptivity made it possible for the style changes to be rapidly accepted which, in turn, resulted in the distinctly higher level of sales for the five years after transition.

Yet another different, more difficult shifting trend problem appears in Figure 3.14 for Process Control Company's (pseudonym) sales. This company manufactures and sells controllers that are used to regulate the operations of manufacturing process equipment. In the operation of plastic molding machinery these controllers govern the different heat and pressure cycles. Characteristically controllers are a durable capital investment type product with extreme business cycle fluctuations in sales. This time series (in Figure 3.14) shows a consistent rise from 1961 through 1966 to a cyclical peak, then the decline in 1967, followed by another cyclical peak in 1969, with a sharp drop in orders during 1970-1971 until the rebound in 1972. As controllers are sold for installation on factory processing machinery used to manufacture various types of products, the 1970-1971 low level of controller sales is interpreted as reflecting the overall decline during these years for manufacturing capital equipment investment.

Obviously, the straight-line trend in Figure 3.15 does not fit the data well at any time. This trend, therefore, is judged not useable.

A second approach initially fit a linear trend from 1961 through 1965, reflecting a consistent upward movement, and then fit a separate trend for 1965 through 1972, as shown in Figure 3.16. The chart shows an almost flat line for 1965-1972 with cyclical peaks in 1966, 1969, and 1972, and lows in 1967 and 1970-1971. From the general economic viewpoint, it is clear that a change in trend takes place about 1965-1966. We hypothesize that sales since 1965 reflect cyclical variations around an approximately level trend. The calculated slope for the linear trend through 1965-1972 shows a slightly positive value. This figure is not statistically different, however, from the deliberate flat trend we expect based on our economic interpretations just discussed.

Notice in Figure 3.16 that the observations from 1961 through 1966 reflect a consistently increasing, absolute amount of increase in each year which may approach a constant percent of increase each year. This constant rate essentially holds true in the trend shown in Figure 3.17 with a logarithmic vertical scale, fitted to 1961 through 1965. Similarly, a flat line has been fitted from 1965 through 1972 at the calculated geometric mean. The calculated slope of a least-squares logarithmic trend through this 1965-1972 data is also slightly positive but not significantly different from the judgment slope (plotted) of zero. Hence, we conclude that the logarithmic trend analysis represented by Figure 3.17 is essentially no improvement over Figure 3.16 results and is not worthy of further effort for forecasting purposes. One value that Figure 3.17 has, however, is the verification of a consistent percentage annual increase for 1965-1972.

3.8 Second-Degree Polynomial Trend

The examples to this point have illustrated three types of nonlinear trends:

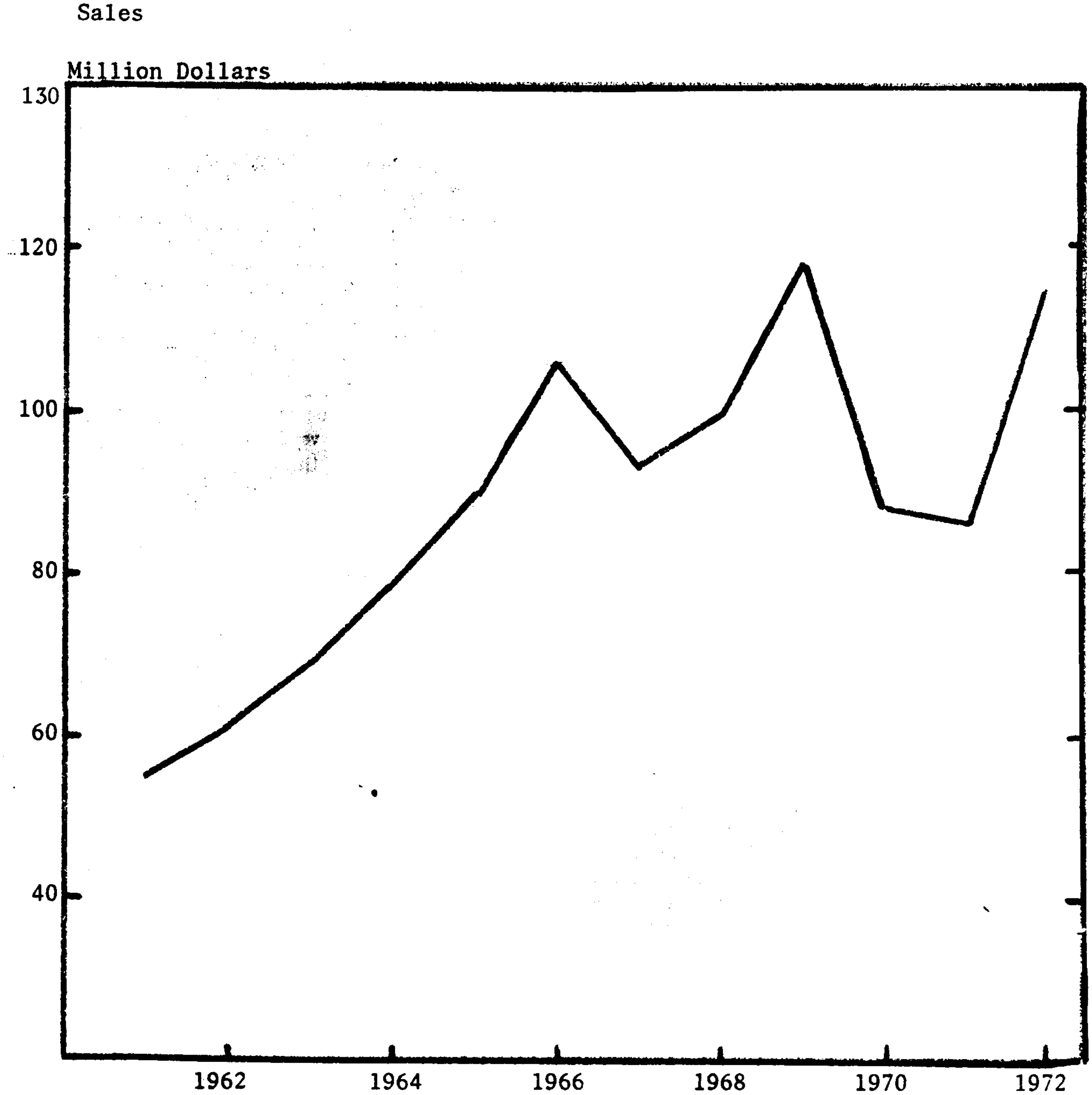
1. Two connecting linear trends.
2. Two linear trends at shifted levels.
3. One logarithmic trend line reflecting gradual increase in absolute amount of change per time unit.

Still further flexibility in fitting curved trends is sometimes necessary, and the second-degree polynomial is a useful tool for this purpose. A curve of this type has only one bend and is symmetrical about this turning point. The curve is always a parabola with varying degrees and directions of curvature. The associated average slope may be either positive or negative, and, when taken together with the amount of curvature, these two characteristics can define a curve which is either everywhere decreasing, increasing through early observations with decreases in the later ones, or conversely, initial decline in data followed by advance.

Figure 3.14

Process Control Company

Sales in Current Dollars

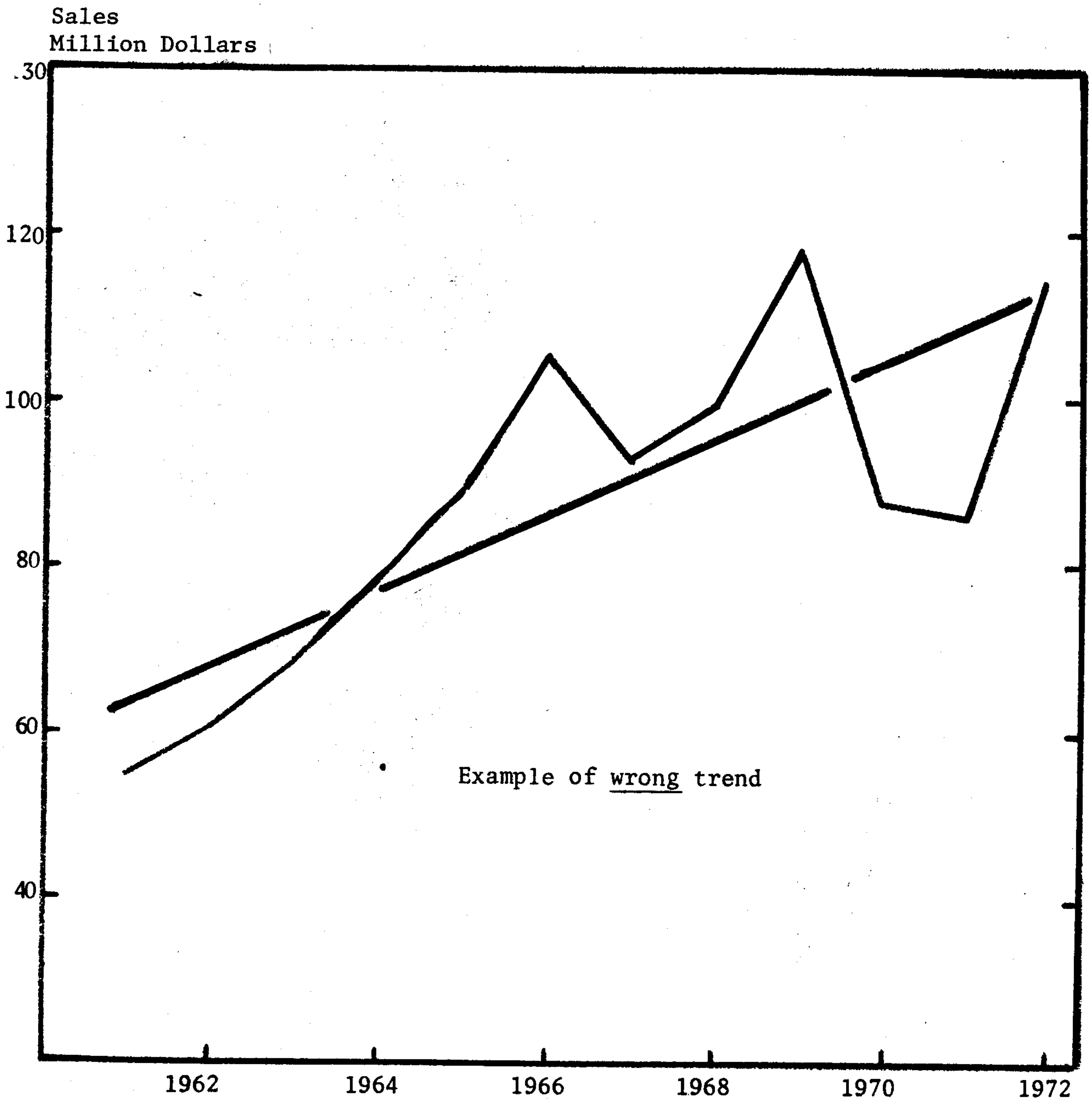


Source: Case study of actual data.

Figure 3.15

Process Control Company

Linear Trend: Least-Squares Method

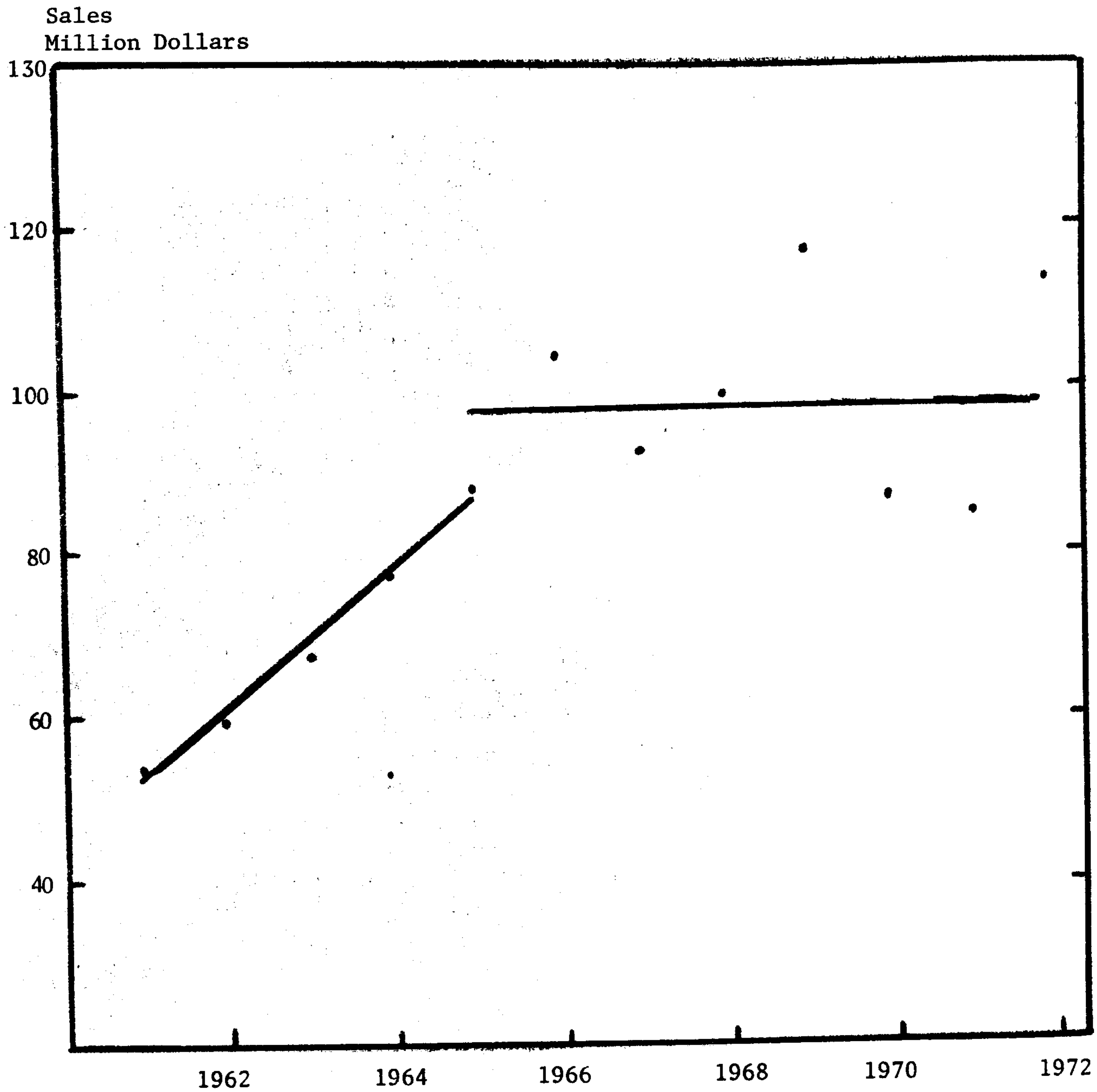


Source: Case study

Figure 3.16

Process Control Company

Two Linear Trends: Least-Squares Method

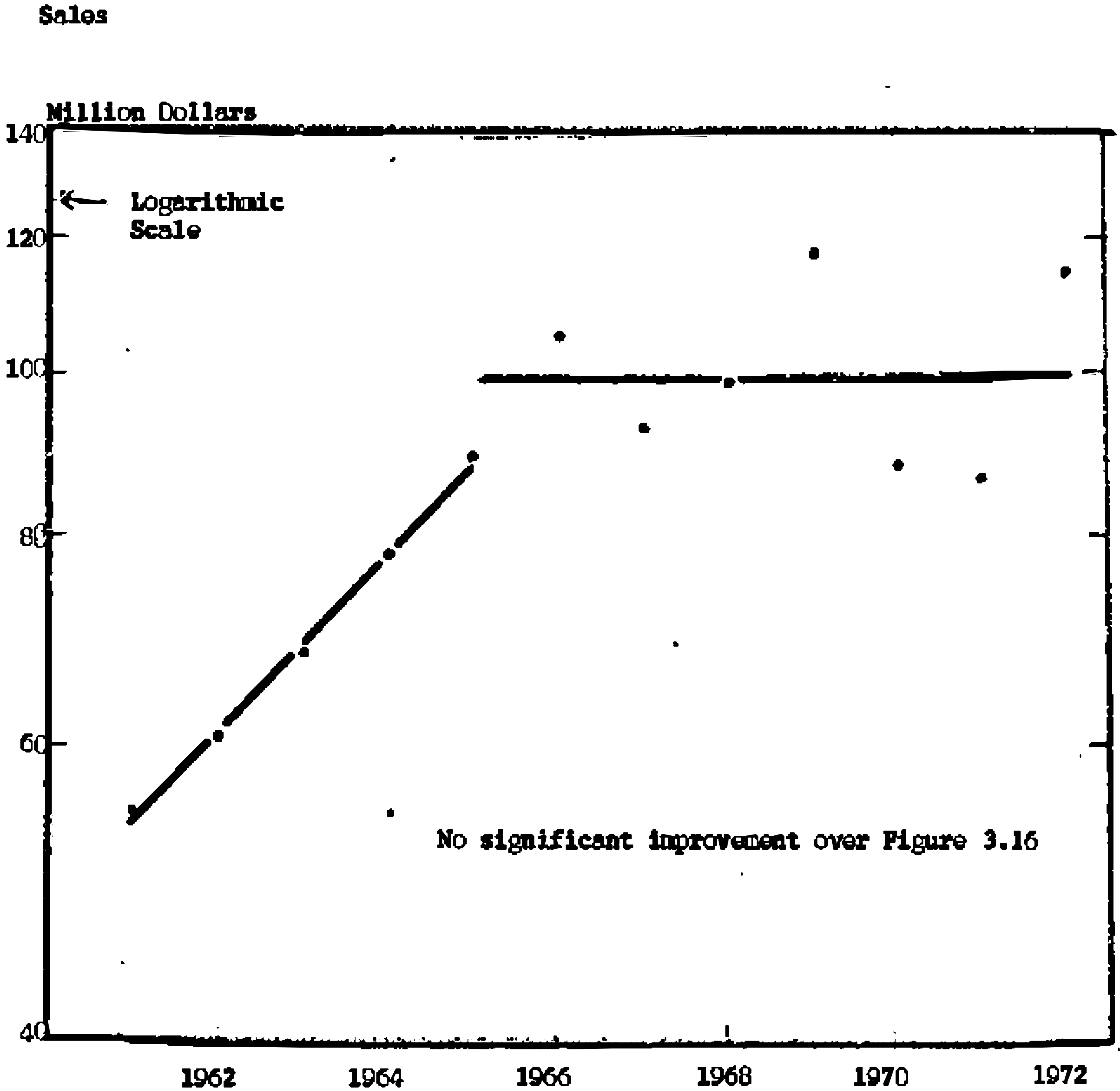


Source: Case study

Figure 3.17

Process Control Company

Two Logarithmic (Exponential) Trends: Least-Squares Method



Source: Case study

Figure 3.18 shows a second-degree polynomial trend fitted to the annual data of Process Control Company. The least-squares statistical criterion of minimum vertical deviations of the actual data from the fitted line suggests that this curve is a moderately good historical fit.

Considering the concept of a second-degree polynomial curve as an explanation of our economic interpretation of the data, we find no intuitive relationship between the formula for a second-degree curve

$$Y_c = b_0 + b_1 X + b_2 X^2 \tag{3.6}$$

and the changing nature of the data. Moreover, extrapolating the fitted second-degree polynomial trend (see Figure 3.18) yields consistently lower values for a forecast than those expected from an analysis of the business cycle, which suggests overall increase in 1973. Thus we reject this trend as not being appropriate to the economic circumstances of the case, despite a moderately good statistical fit to the historical data.

The next case-study series, for industrial wheel tractor industry retail sales, is in physical *units* and, therefore, does not directly include price inflation as do the previous cases containing dollar sales data. These tractor units historically have increased very little in average horse power because of their nature as utility vehicles, primarily serving as a power supply and mount for front-end loaders, rear-mounted backhoes, center-mounted mowers, and various other attachments. As a result of their stable nature, industrial wheel tractors have been moderately consistent in size and type over the years in the series.

Figure 3.19 shows annual unit sales by all manufacturers in this industry. The trend, while irregular, is moderately rising with cyclical peaks in 1966 and 1969 and the record high in 1972.

Figure 3.20 shows a least-squares linear trend fitted to the tractor-industry data with fairly satisfactory results. The three high points appear as deviations above the trend, and the years 1967 and 1971 show as deviations below the trend. An extrapolation of this linear trend to 1973 implies a decrease from 1972, which is perceived as inappropriate at the time of this writing, based on the rise in capital expenditure survey expectations for 1973 and the actual increases experienced during the early months of 1973.

Figure 3.21 shows a second-degree polynomial trend fitted to the tractor unit series. This curve exhibits only a slight bend and is a moderately good statistical fit for the historic period. Extrapolating the second-degree trend reflects an increasing amount of annual increase. This trend extrapolation might provide a reasonable forecast for one or two years ahead, but this is not consistent with the long-term outlook for the industrial wheel tractor industry and, consequently, must be rejected as a forecast.

Hand calculations for the second-degree polynomial trend are illustrated in Table 3.2 for the data on industrial wheel tractors. This calculation also uses a centered origin, but here it is midway between two years, 1967 and 1968. Notice that x now represents one-half year and 2x is needed to show a full year in time. Similar results were obtained using a terminal computer program which correlates a coded x with the dependent Y variable.

Although only one turn, possibly a very mild one, can occur for a second-order polynomial, higher degree polynomials produce more turns. Polynomial trend equations are of the general form:

$$Y_c = a + bX + cX^2 + dX^3 + \ldots + jX^n \tag{3.7}$$

A straight line is said to be of the "first degree." When the term cX^2 includes the highest exponent for X, the equation (see Equation 3.16) produces a "second-degree" polynomial having one bend. Correspondingly, if X is carried to the cubic power, dX^3, we have a "third-degree" curve which changes direction twice, and a fourth-degree curve with eX^4 turns three times.

While it is possible to obtain a polynomial trend equation which fits data quite well by increasing the degree of the curve, there is no actual advantage to doing so in sales forecasting because the resulting equation would be describing cyclical or possible erratic fluctuations instead of trend. Thus as a matter of practicality we suggest that polynomial curves more complex than the second degree should seldom be used to describe trend.

3.9 Second-Degree Logarithmic Trend

Occasionally, you may encounter data which, when graphed on semilogarithmic paper, persist in exhibiting curvature of one bend. Figure 3.22 illustrates such a case for commercial poultry production in the United States during 1955-1972. This time series is increasing at a decreasing rate, a frequently encountered situation which reflects approaching market saturation for the poultry industry. We have fitted a second-degree logarithmic trend to the data using:

$$\log Y_c = \log a + X \log b + X^2 \log c \tag{3.8}$$

The computing procedure is equivalent to fitting a second-degree polynomial curve to the logarithms of the poultry production levels. We also have included in Figure 3.22 the extrapolation of the trend curve, which appears to provide reasonable projections for poultry production through 1976.

3.10 Asymptotic Growth Trends

The linear and nonlinear curves described thus far provide excellent fits to many types of time series. However, such trend equations are frequently inadequate to describe certain classes of time series, such as those for product life cycles. The so-called "growth trends" which have upper and/or lower asymptotes are then more appropriate. Two of the more important types of growth curves are the modified exponential and Gompertz curve.

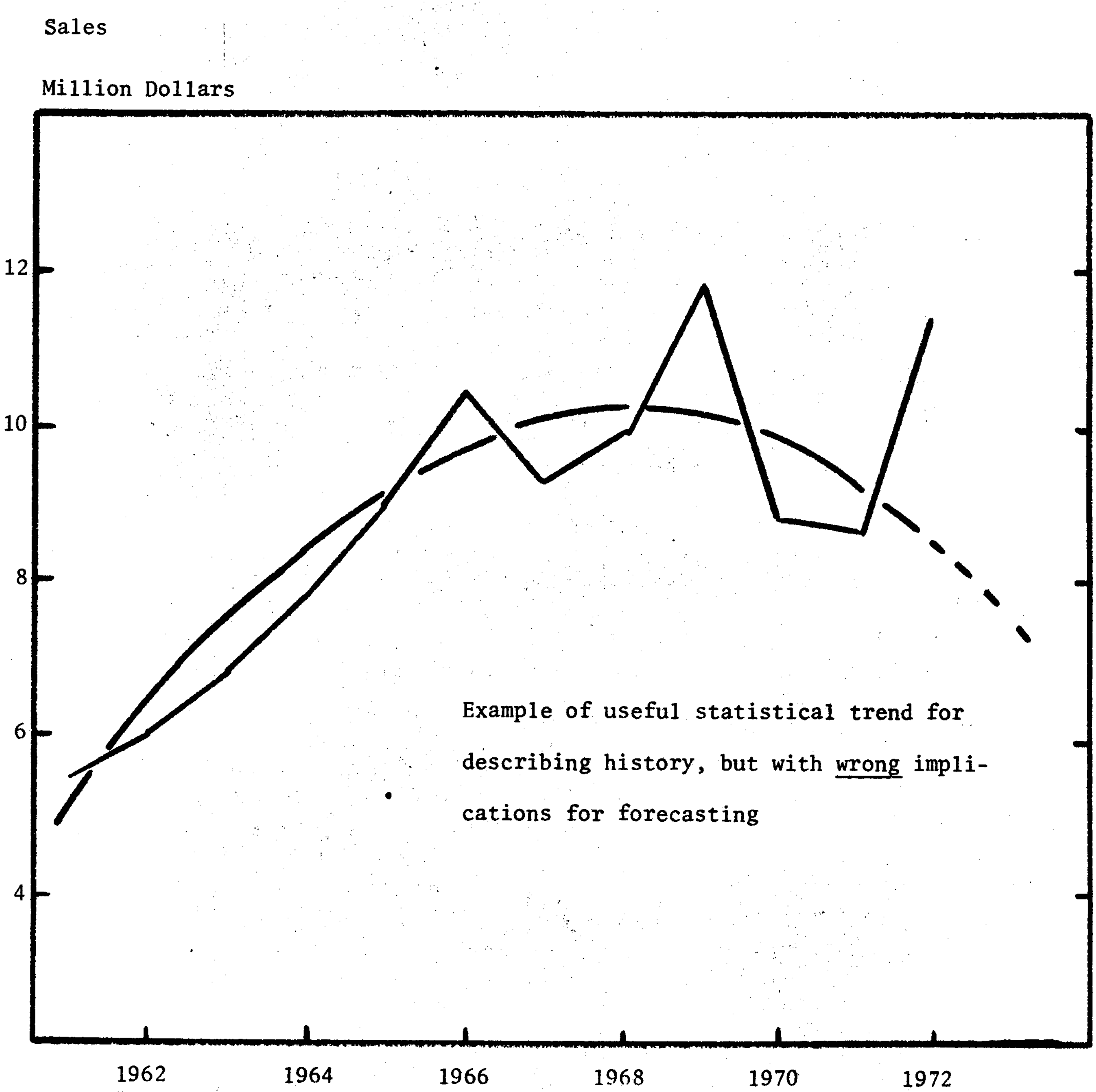
For the *modified exponential* trend, the amount of growth or decline diminishes at a constant percentage rate per time unit, with the curve approaching an upper limit. As illustrated in Panel A of Figure 3.23 our attention (with regard to describing sales data) is given primarily to increasing series, but this equation will also fit decreasing series.

The *Gompertz curve* describes a trend in which growth increments of the logarithms of sales data are declining at a

Figure 3.18

Process Control Company

Second-Degree Polynomial (Parabolic) Trend: Least-Squares Method

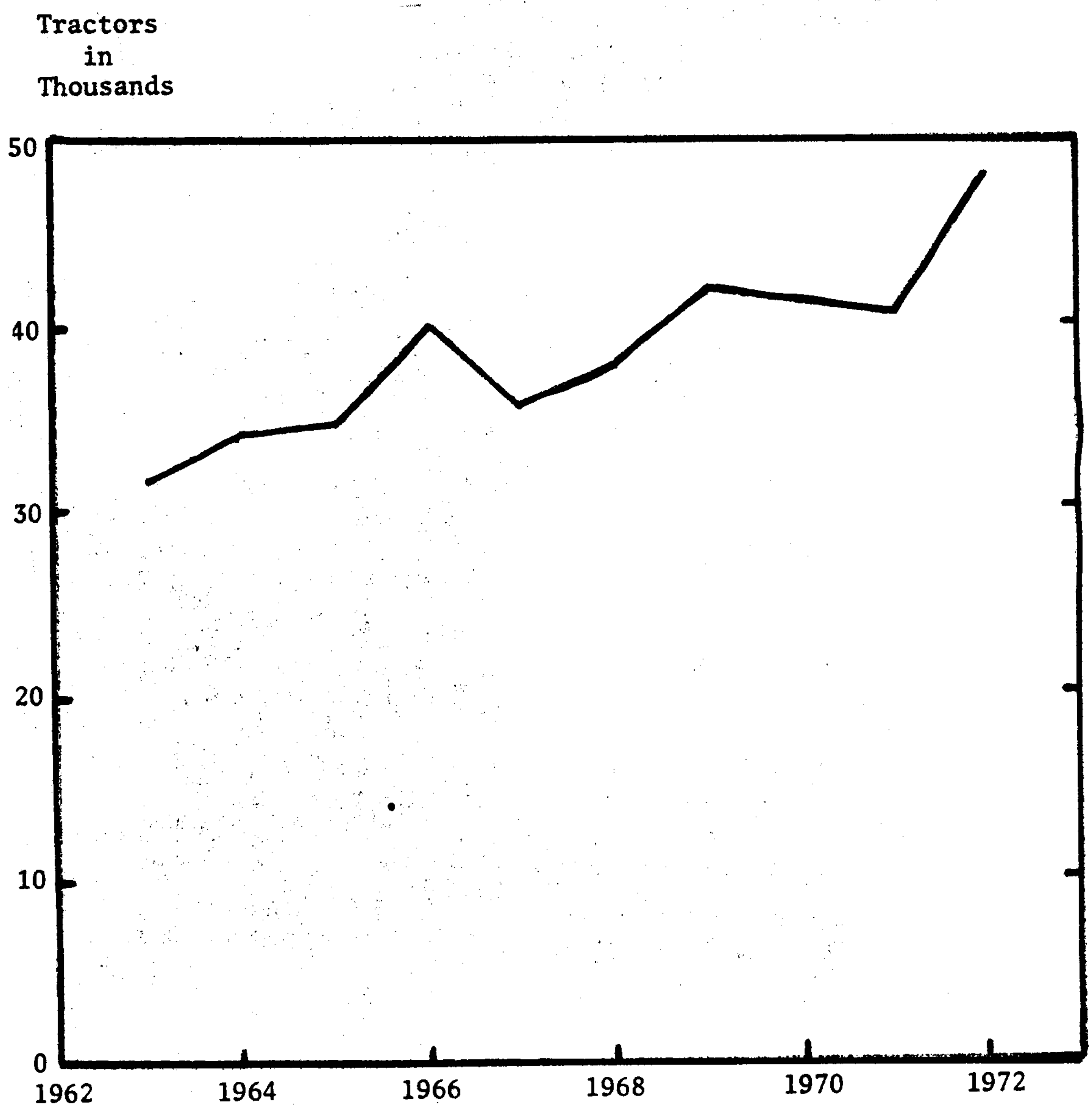


Source: Case study of actual data.

Figure 3.19

Industrial Wheel Tractor

Industry Unit Retail Sales, United States



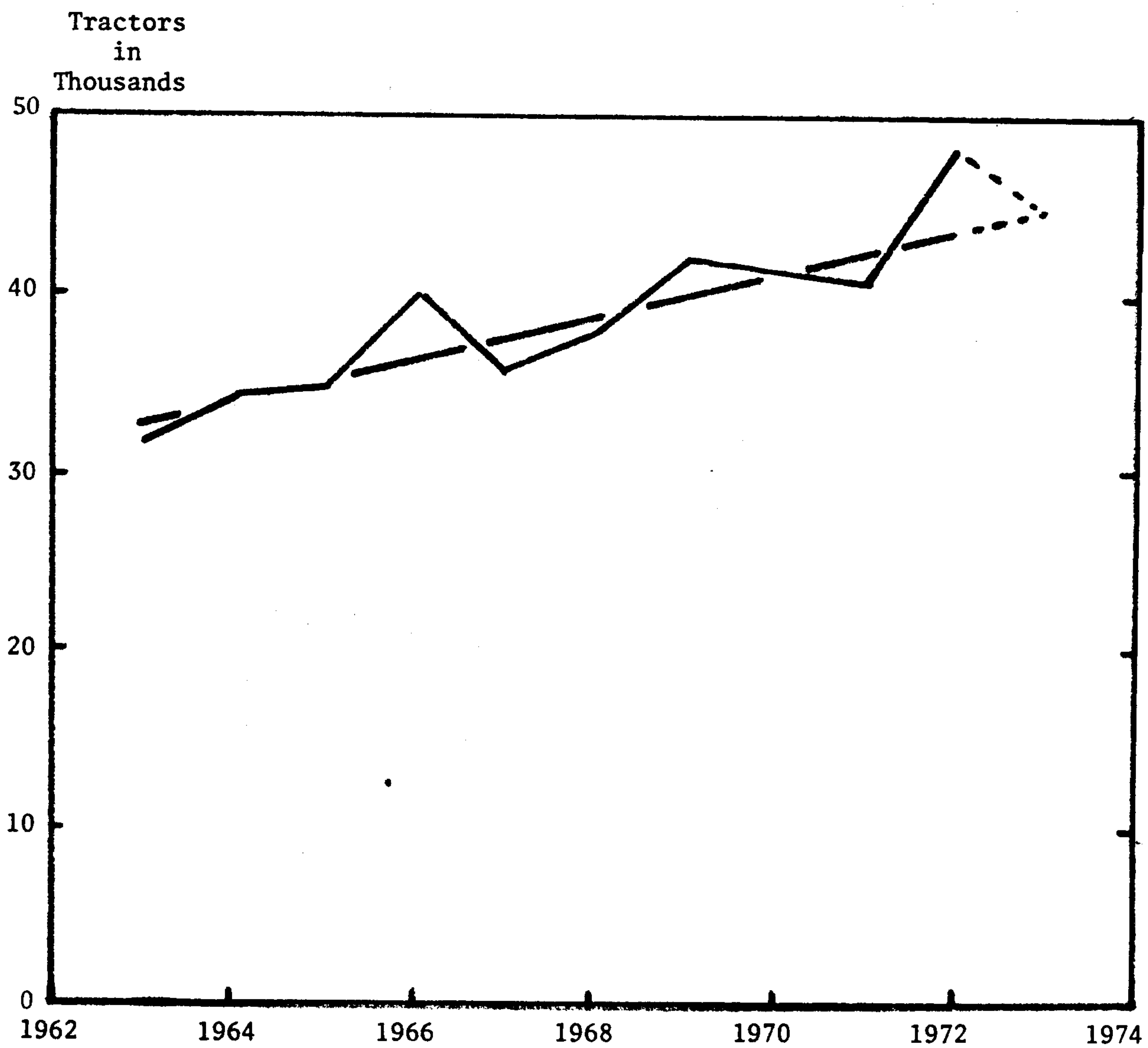
Source: Farm and Industrial Equipment Institute

Figure 3.20

Industrial Wheel Tractor

Industry Unit Retail Sales, United States

Linear Trend: Least-Squares Method



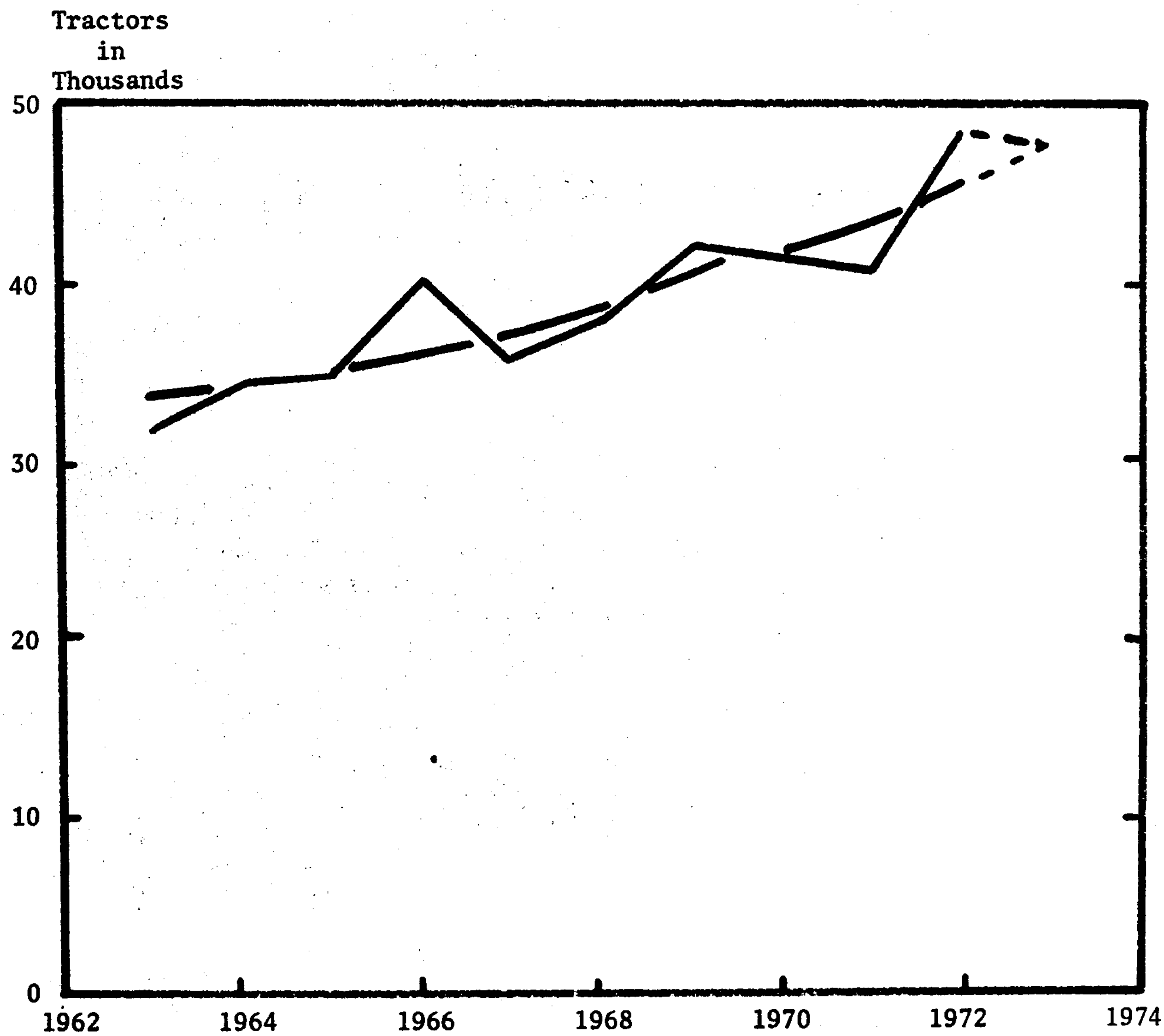
Source: Farm Industrial Equipment Institute.

Figure 3.21

Industrial Wheel Tractor

Industry Unit Retail Sales, United States

Second-Degree Polynomial Trend: Least-Squares Method



Source: Farm Industrial Equipment Institute

Table 3.2

Second Degree Polynomial Trend: Least-Squares Method

Industrial Wheel Tractor

Industry Unit Retail Sales, United States

Year	Y	x	x^2	xY	x^2Y	x^4	a+bx	cx^2	Y_c
1963	31.2	-9	81	-280.8	2,527.2	6,561	31.49	.97	32.5
1964	34.0	-7	49	-238.0	1,666.0	2,401	32.94	.59	33.5
1965	34.7	-5	25	-173.5	867.5	625	34.40	.30	34.7
1966	39.4	-3	9	-118.2	354.6	81	35.86	.11	36.0
1967	35.6	-1	1	- 35.6	35.6	1	37.31	.01	37.3
1968	37.7	+1	1	37.7	37.7	1	38.77	.01	38.8
1969	41.7	+3	9	125.1	375.3	81	40.22	.11	40.3
1970	41.1	+5	25	205.1	1,027.5	625	41.68	.30	42.0
1971	40.9	+7	49	286.3	2,004.1	2,401	43.13	.59	43.7
1972	48.0	+9	81	432.0	3,888.0	6,561	44.59	.97	45.6
	384.3	0	330	+240.1	12,783.4	19,338			384.4
	ΣY	Σx	Σx^2	ΣxY	Σx^2Y	Σx^4			ΣY_c

These normal equations must be solved:

I. $\Sigma Y = an + c\Sigma x^2$

II. $\Sigma xY = b\Sigma x^2$

III. $\Sigma x^2Y = a\Sigma x^2 + c\Sigma x^4$

From equation II, $b = \dfrac{\Sigma xY}{\Sigma x^2} = \dfrac{240.1}{330} = 0.7276$

$$
\begin{array}{lrcrcr}
\text{I} & 384.3 & = & 10a & + & 330c \\
\text{III} & 12{,}783.4 & = & 330a & + & 19{,}338c \\
-33\ (\text{I}) = & -12{,}681.9 & = & 330a & - & 10{,}890c \\
\hline
 & 101.5 & = & & & 8{,}448c
\end{array}
$$

$$\frac{101.5}{8{,}448.0} = c = .012015$$

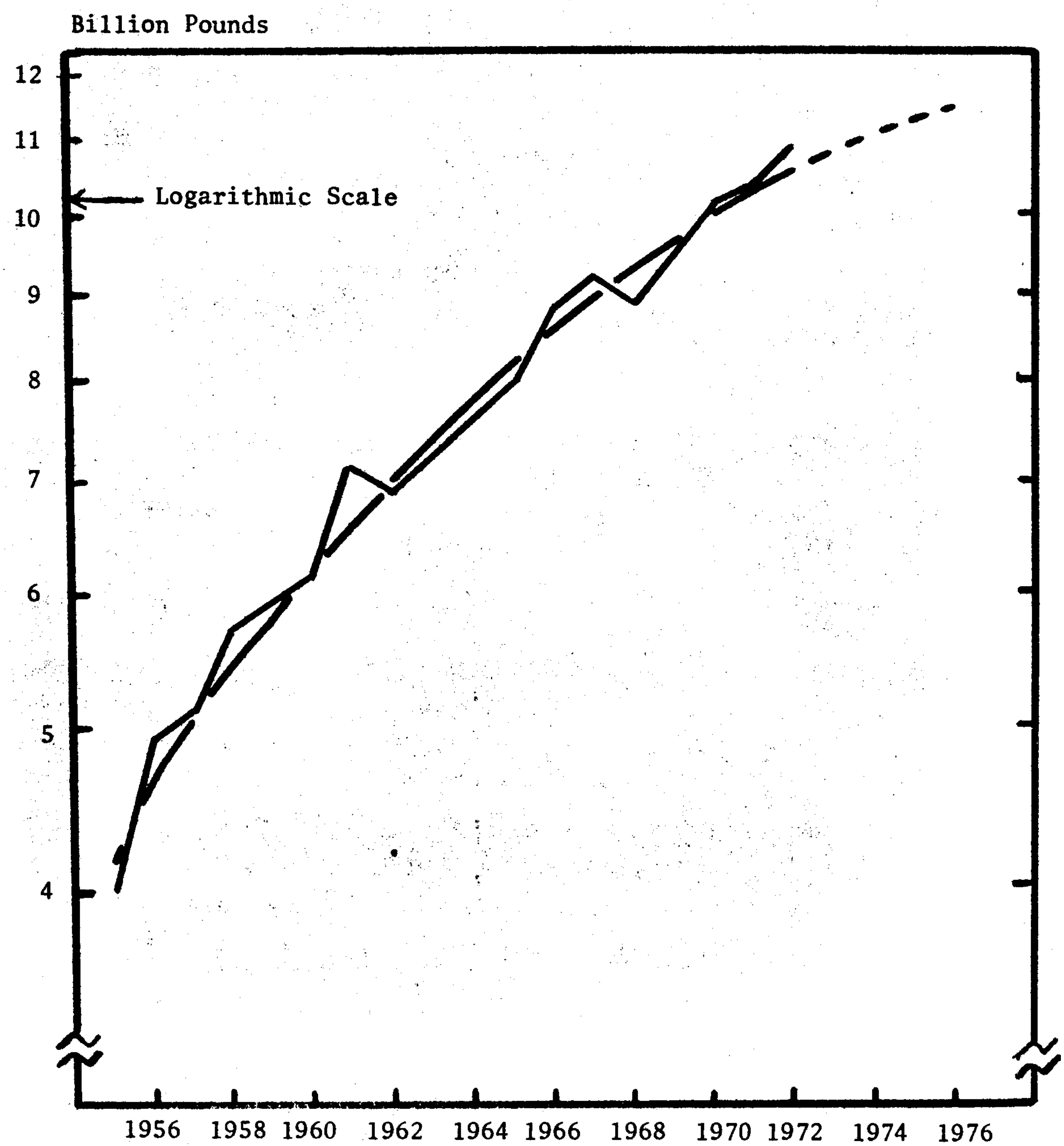
$$
\begin{aligned}
384.3 &= 10a + 330(.012015) \\
380.34 &= 10a \\
38.034 &= a
\end{aligned}
$$

$$Y_c = 38.034 + 0.7276\,x + .012015x^2$$

Source: Farm Industrial Equipment Institute

Figure 3.22

Poultry: U. S. Commercial Production

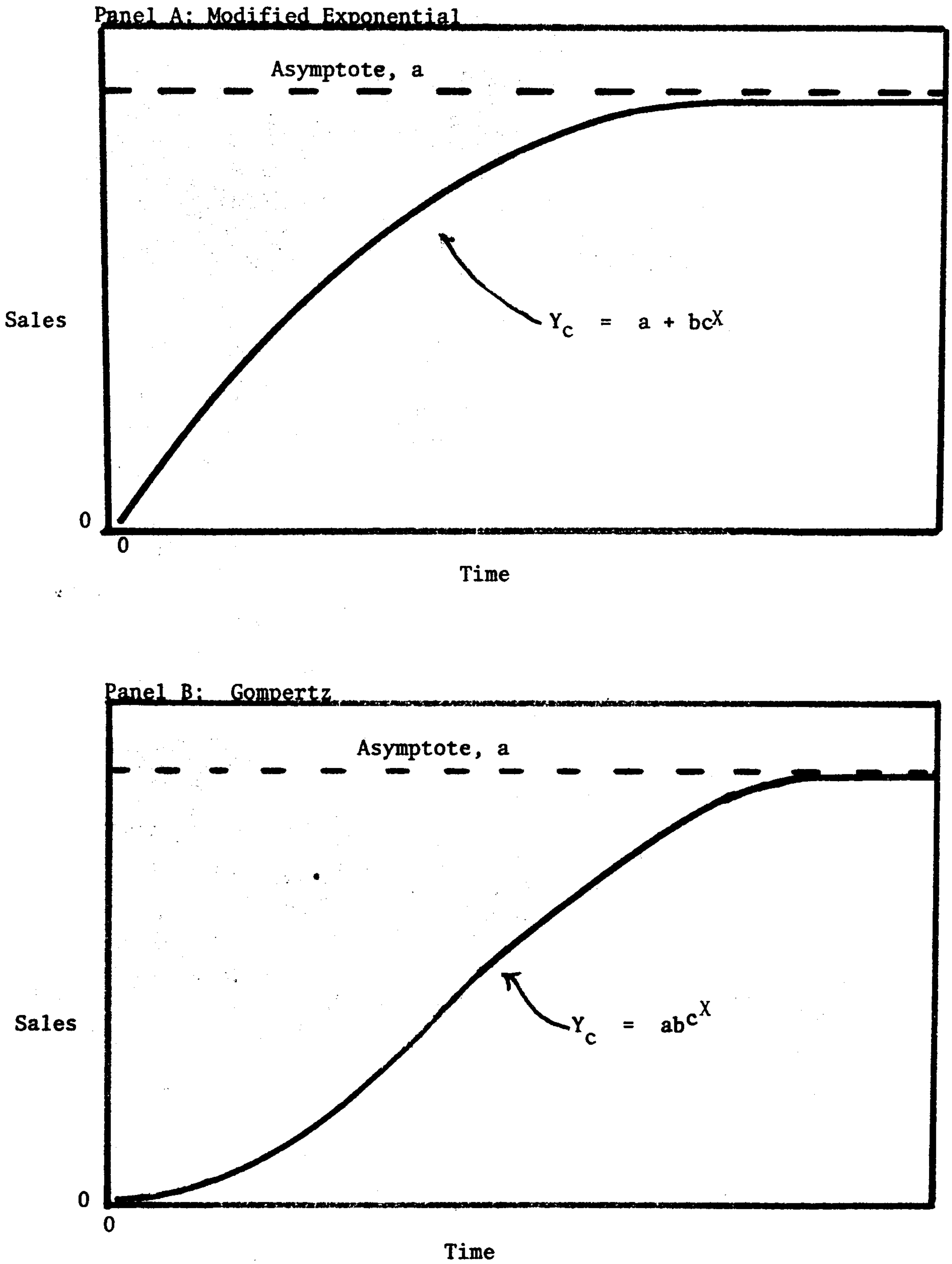


Source: U. S. Department of Agriculture.

Figure 3.23

General Forms of the

Modified Exponential and Gompertz Curves



constant percentage rate with time. Hence, the Y_c values show a diminishing ratio of advance, although the ratio does not decline either by a constant percentage rate or by an absolute amount. The shape of the Gompertz equation on which our major interest centers in forecasting sales is shown in Panel B of Figure 3.23. This curve has a calculated upper asymptote and a lower asymptote of zero.

The Gompertz curve is a conceivable alternate equation for the trend in Style Shop's sales data. (See Figure 3.11.) Both the economic environment and the sales data, however, do not properly adhere to the underlying requirements for the Gompertz curve, which reflects a slow start, rapid growth, then a leveling off, against some upper limit. The Gompertz curve is more fundamentally applicable to a product life cycle, whereas men's wear has been a steady commodity in our economy for many decades. Hence, we reject the Gompertz trend as a good descriptive device for the Style Shop data.

Since the use of growth curves more appropriately describes trends in long-term series, further treatment is deferred to Chapter 13.

3.11 Critical Importance of Trend Analysis in Forecasting

All trends are "time trends" since the only causal or independent force in the trend is the passage of time. The implicit economic or business causal philosophy for using a time trend as a method of making a forecast, therefore, is based on these two concepts:

1. Knowledgeable persons in the field may believe that the combination of forces acting upward and downward on the sales trend in the next year or so may average out to be about the same net influence as in the average of the past few years. If this is true, an extrapolation of a slowly changing time trend may be a perfectly good forecasting device.
2. If there is no strong perception that influences on sales will be substantially different from the average of past influences for change, the least-squares time trend represents a valid statistical, mean-type projection. This quantitatively based projection has the advantage over purely subjective prognostication in that it is logically reproducible and is free from the biases of judgment forecasts. Our experience has confirmed a number of cases in which simple time trend forecasts by the method of least-squares would have been much better than the "expert" projections made by company executives with personal biases arising largely from the business "atmosphere" of the few weeks preceding the date of making annual forecasts.

Before concluding our discussion of trend analysis, we point out the fundamental limitation of a time trend, namely, that the projected curve in no way reflects the influences of factors in an industry or market which differ substantially from the average of past periods. Thus the only way to alter a least-squares trend forecast is by judgment modification. We might say, for example, that, as a result of subjective assessment, estimated sales next year will be 10% above the projection of the long-term sales trend. This conjecture may be legitimate, provided causal influences exist and can be perceived as such. An adjusted forecast of this type is usually better than a forecast of no deviation from the least-squares trend in cases where changes are strongly felt to be present but which cannot, at the current stage of development for the forecasting system, be reflected numerically. Accordingly, we note that subjective adjustment of a least-squares trend projection based on cause generally provides better results than does forecasting founded exclusively on "informed intuition."

Footnotes

1. Equation 3.1 has been shortened from the full notation: where "i" = month or quarter and "j" = year, $O_{ij} = X_{ij} \times T_{ij} \times S_{ij} \times C_{ij} \times I_{ij}$. If a constant seasonal component (see Chapter 4) applies, the S_{ij} shortens to S_i.

2. Some time series may require an additive model of the form: $O = T + S + C + I$. See Clark and Schkade, *Statistical Analysis for Administrative Decisions*, 2nd ed. (Cincinnati, South-Western Publishing Co., 1974), p. 643.

3. Annual data do not have seasonal influences, by definition.

4. Semilogarithmic graph paper has a logarithmic vertical scale and an arithmetic horizontal scale.

5. Notice, if we take the antilog of both sides of Equation 3.5 we obtain the exponential form: $Y_c = ab^x$, which is not suited for directly applying the least-squares fit procedure.

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